MA 16020 Lesson 25: Lagrange multipliers I

Constrained min, max: Given a function $z = f(x, y)$ of two variables, sometimes it is is desirable to find the mionimum or maximum of f only among certain subset of the xy-plane, namely among all the points (x, y) satisfying certain equation (*constraint*)

$$g(x, y) = C$$

Example: Find the closest point of the curve $xy = 1$ to the origin.

To solve problems of this sort, we again want to find critical points, for which we use a special version of first derivative test called *method of Lagrange multipliers.*

When trying to minimize/maximize the value of the function $z = f(x, y)$ subject to the constraint $g(x, y) = C$, the critical points are given as points (x, y) that are solutions to the system of the equations:

as well as the original constraint:

typical strategy (not always):

Exercise 1. Finishing Example from previous page, i.e. find the closest point of the curve $xy = 1$ to the origin.

Exercise 2. Find the maximum of the function $f(x, y) = 8x^2 - 2y$ subject to the constraint $x^2 + y^2 = 4$.

Exercise 3. Find the point(s) (x, y) where the function $f(x, y) = \ln(3xy^2)$ attains maximal value, subject to constraint $7x^2 + y^2 = 21$.

Exercise 4. Find the minimal value of the function $f(x, y) = y^3 e^{x^2}$ subject to the constraint $10x^2 - 3y = 8$.