MA 16020 Lesson 35: Eigenvalues and eigenvectors I

The **eigenvector** of a square ($n \times n$) matrix A is a 1-column matrix (i.e. *vector*) $v \neq 0$ such that:

In such case, we call λ the *eigenvalue* of A.

Example: Let us check whether the vector $\begin{bmatrix} 3 \\ 0 \\ 1 \end{bmatrix}$ is an eigenvector for the matrix $\begin{bmatrix} 1 & -1 & 3 \\ -1 & 2 & 3 \\ 2 & 4 & -4 \end{bmatrix}$, and if yes, find the eigenvalue.

How to find an eigenvalue. To find eigenvectors of a matrix, it is practical to find the possible eigenvalues λ first.

Example: Find all the eigenvalues of the matrix $\begin{bmatrix} 2 & -3 \\ -1 & 4 \end{bmatrix}$.

Suppose that the eigenvalue is λ and the eigenvector is $\begin{bmatrix} x \\ y \end{bmatrix}$. Then we have:

We are looking for λ's such that the equation above has **nonzero** solution $\begin{bmatrix} x \\ y \end{bmatrix}$. This means that:

To check this, we employ the determinant:

Exercise. Find all the eigenvalues of the matrix $\begin{bmatrix} 3 & 1 \\ 2 & 4 \end{bmatrix}$.

How to find the eigenvectors. Finally, let us continue the original example (matrix $\begin{bmatrix} 2 & -3 \\ -1 & 4 \end{bmatrix}$) to find eigenvectors to the found eigenvalues:

Finding eigenvalues and eigenvectors - summary.

1. Set up the *characteristic equation*:

2. Eigenvalues of A are then obtained as:

3. For each eigenvalue λ, the corresponding eigenvectors are obtained as:

Example: Find the eigenvalues and eigenvectors for the matrix $\begin{bmatrix} 1 & 5 \\ 1 & 3 \end{bmatrix}$.

Example: Find the eigenvalues and eigenvectors for the matrix $\begin{bmatrix} 7 & -4 \\ 4 & -1 \end{bmatrix}$.