MATH 16020 Lesson 6: Separation of Variables I

Spring 2021

Definition. A **differential equation (DE)** is _____

_____.

Examples:

Definition. A DE is **separable** if _____

Example. Show $\dfrac{dy}{dx} = x^3 e^{y-x^4}$ is separable.

Definition. A **solution** to a DE is _____

A **particular solution** to a DE is _____

Example. A particular solution to $\dfrac{dy}{dx} = 3y$ is $y(x) = 2e^{3x}$ as shown below:

Example 1. Solve $y' = ky$ if $y(0) = 6$ and $y'(0) = 12$.

Example 2. Solve the differential equation below where $y = 2$ if $t = 1$.

$$\frac{dy}{dt} = \frac{\ln(t)}{3y}$$

Example 3. Write a differential equation describing each of the following types of proportionality:

1. A strain of bacteria grows at a rate (directly) proportional to its population P at time t.

2. A strain of bacteria grows at a rate inversely proportional to its population P at time t.

3. The rate at which a group of 8300 people become infected is jointly proportional to the number of people already infected P (at time t) and the people not infected.

Example 4. A radioactive element has a half-life of 5 years. If the element initially weighs 4 pounds, find the amount left after 12 years.

Example 5. After 10 minutes in Joe's room, his tea has cooled from 100^oC to 50^oC. If the room temperature is 20^oC, find the temperature 50 minutes later. Round to the nearest hundreth.