Quiz 1 Solution

1. Find all m such that the function $\varphi(x) = x^m$ is a solution to the equation

$$2x^2 \frac{\mathrm{d}^2 y}{(\mathrm{d}x)^2} - 3x \frac{\mathrm{d}y}{\mathrm{d}x} + 2y = 0 \ .$$

Solution: For $y = x^m$ we have

$$\frac{\mathrm{d}y}{\mathrm{d}x} = mx^{m-1}, \quad \frac{\mathrm{d}^2 y}{(\mathrm{d}x)^2} = m(m-1)x^{m-2},$$

so plugging into the diff. equation yields

$$\begin{aligned} 2x^2 \cdot m(m-1)x^{m-2} - 3x \cdot mx^{m-1} + 2x^m =& 0 \\ 2m(m-1)x^m - 3mx^m + 2x^m =& 0 \\ (2m(m-1) - 3m + 2)x^m =& 0 \\ 2m(m-1) - 3m + 2 =& 0 \\ 2m^2 - 5m + 2 =& 0. \end{aligned}$$

The discriminant of the resulting quadratic equation is $25 - 4 \cdot 2 \cdot 2 = 9$, and so the two solutions to this equation are

$$m = \frac{5 \pm \sqrt{9}}{2 \cdot 2} = 2, \ \frac{1}{2}.$$

Therefore the desired exponents m are $m = 2$ and $m = 1/2$.