Quiz 5 Solution

1. Compute the inverse of the matrix

$$\begin{bmatrix} 1 & 3 & 0 & 3 \\ -1 & -2 & -2 & 1 \\ 2 & 6 & 1 & 4 \\ 2 & 4 & 4 & -1 \end{bmatrix}.$$

Solution: We use the method from the lecture, i.e. by row operations we transform A into the identity matrix and keep track of the used operations on a identity matrix, which will transform into the inverse matrix we want. We have

$$\left[\begin{array}{cccc|cccc} 1 & 3 & 0 & 3 & 1 & 0 & 0 & 0 \\ -1 & -2 & -2 & 1 & 0 & 1 & 0 & 0 \\ 2 & 6 & 1 & 4 & 0 & 0 & 1 & 0 \\ 2 & 4 & 4 & -1 & 0 & 0 & 0 & 1 \end{array}\right] \sim_1 \left[\begin{array}{cccc|cccc} 1 & 3 & 0 & 3 & 1 & 0 & 0 & 0 \\ 0 & 1 & -2 & 4 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & -2 & -2 & 0 & 1 & 0 \\ 0 & -2 & 4 & -7 & -2 & 0 & 0 & 1 \end{array}\right] \sim_2$$

$$\left[\begin{array}{cccc|cccc} 1 & 3 & 0 & 3 & 1 & 0 & 0 & 0 \\ 0 & 1 & -2 & 4 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & -2 & -2 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 2 & 0 & 1 \end{array}\right] \sim_3 \left[\begin{array}{cccc|cccc} 1 & 3 & 0 & 0 & 1 & -6 & 0 & -3 \\ 0 & 1 & -2 & 0 & 1 & -7 & 0 & -4 \\ 0 & 0 & 1 & 0 & -2 & 4 & 1 & 2 \\ 0 & 0 & 0 & 1 & 0 & 2 & 0 & 1 \end{array}\right] \sim_4$$

$$\left[\begin{array}{cccc|cccc} 1 & 3 & 0 & 0 & 1 & -6 & 0 & -3 \\ 0 & 1 & 0 & 0 & -3 & 1 & 2 & 0 \\ 0 & 0 & 1 & 0 & -2 & 4 & 1 & 2 \\ 0 & 0 & 0 & 1 & 0 & 2 & 0 & 1 \end{array}\right] \sim_5 \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 10 & -9 & -6 & -3 \\ 0 & 1 & 0 & 0 & -3 & 1 & 2 & 0 \\ 0 & 0 & 1 & 0 & -2 & 4 & 1 & 2 \\ 0 & 0 & 0 & 1 & 0 & 2 & 0 & 1 \end{array}\right]$$

(where
$\sim_1$=adding suitable multiples of the first row to the others to clear the first column,
$\sim_2$=adding the first row to the third 2 times,
$\sim_3$=adding suitable multiples of the fourth row to the others to clear the fourth column,
$\sim_4$=adding the third row to the second 2 times,
$\sim_5$=subtracting the second row from the first 3 times).

Thus, we see that the inverse matrix to the given matrix is the matrix

$$\begin{bmatrix} 10 & -9 & -6 & -3 \\ -3 & 1 & 2 & 0 \\ -2 & 4 & 1 & 2 \\ 0 & 2 & 0 & 1 \end{bmatrix}.$$