Quiz 6 Solution

1. Compute the determinant, the adjugate matrix and the inverse matrix of the matrix

$$A = \begin{bmatrix} -2 & 3 & 0 \\ 0 & -2 & -1 \\ 2 & 0 & 2 \end{bmatrix}.$$

Solution: Let us start with the adjugate matrix. It is given as the transpose of the cofactor matrix C. By direct computation, we get:

$$C = \begin{bmatrix} -4 & -2 & 4 \\ -6 & -4 & 6 \\ -3 & -2 & 4 \end{bmatrix}$$

(For example: the $(1,1)$-entry of C is obtained as $(-1)^{1+1}$ times the subdeterminant obtained by deleting first row and column from A, which is $\begin{vmatrix} -2 & -1 \\ 0 & 2 \end{vmatrix} = (-2) \cdot 2 - (-1) \cdot 0 = -4$, the $(2,1)$ entry is $(-1)^{2+1} \begin{vmatrix} 3 & 0 \\ 0 & 2 \end{vmatrix} = -6$, and so on.). Now the adjugate matrix is

$$\mathrm{Adj}(A) = C^T = \begin{bmatrix} -4 & -6 & -3 \\ -2 & -4 & -2 \\ 4 & 6 & 4 \end{bmatrix}.$$

To compute the determinant, we can either compute it directly, use row operations to simplify the matrix, or use cofactor expansion. Let's use cofactor expansion since we have already computed the cofactors (and recorded them as entries in C).

Expanding the first row of A, we have

$$\det A = (-2) \cdot (-4) + 3 \cdot (-2) + 0 \cdot 4 = 8 - 6 = 2.$$

Finally, the inverse matrix A^{-1} is obtained from the adjugate matrix and the determinant by the formula

$$A^{-1} = \frac{1}{\det(A)} \cdot \mathrm{Adj}(A) = \frac{1}{2} \begin{bmatrix} -4 & -6 & -3 \\ -2 & -4 & -2 \\ 4 & 6 & 4 \end{bmatrix} = \begin{bmatrix} -2 & -3 & -\frac{3}{2} \\ -1 & -2 & -1 \\ 2 & 3 & 2 \end{bmatrix}.$$