(11) distance from Graph to point problems means optimizing distance Formula.

$$d = \sqrt{(x-3)^2 + (y-2)^2}$$

objective

$$D = d^2 = (x-3)^2 + (y-2)^2$$

Constraint $\quad y = \sqrt{x} + 2$

plug constraint into objective, take Derivative

$$D = (x-3)^2 + \left((\sqrt{x}+2)-2\right)^2$$

$$= x^2 - 6x + 9 + x = x^2 - 5x + 9$$

$D' = 2x - 5 = 0$ set

Find C.N. $\quad 2x = 5 \longrightarrow x = \frac{5}{2}$ E

$D'' = 2 > 0$

objective : cost (based off S.A.)

constraint: Volume : $16 = w^2 h$

total cost = (cost of ~~metal~~ top+bottom) (Area of top+Bottom)

+ (cost of sides) Area of sides)

$C = \$1(2w^2) + \$4(4wh)$

top+bottom

4 sides

$C = 2w^2 + 16wh$

solve for h in constraint: $h = \frac{16}{w^2}$

$C = 2w^2 + 16w\left(\frac{16}{w^2}\right)$

$C = 2w^2 + \frac{16^2}{w}$

Optimize

$C' = 4w - \frac{16^2}{w^2} = 0$

C.N: $4w = \frac{16^2}{w^2}$

$w^3 = \frac{16^2}{4} = 64$

$w = (64)^{1/3} = 4$

Plug C.N. $w = 4$ which is rel/abs min

$C(4) = 32 + \frac{256}{4} = 32 + 64 = 96$

option

2

(58) $y'' = 2 + 4e^x \qquad y'(0) = 1$ and $y(0) = 4$

$y' = \int (2 + 4e^x)\,dx = 2x + 4e^x + C$

$y'(0) = 2(0) + 4e^0 + C = 1$

$4 + C = 1 \rightarrow C = -3$

$y' = 2x + 4e^x - 3$

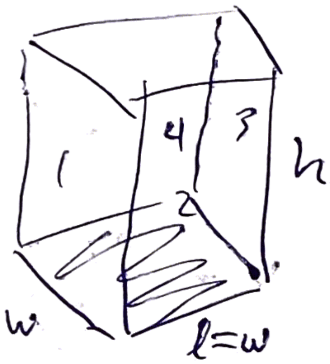
$y = \int (2x + 4e^x - 3)\,dx = x^2 + 4e^x - 3x + C$

$y(0) = (0)^2 + 4e^0 - 3(0) + C = 4$

$4 + C = 4$, so $C = 0$

$y = x^2 + 4e^x - 3x$ (A)

9, 11, 57, 51, 12



(9) Constraint: Surface Area/ Area

objective: volume $V = w^2 h$

$$48 = \underset{\substack{\uparrow \\ \text{bottom}}}{w^2} + \underset{\substack{\uparrow \\ \text{4 sides}}}{4wh} \longrightarrow 48 - w^2 = 4wh$$

$$h = \frac{48 - w^2}{4w}$$

plug h into objective

$$V = w^2\left(\frac{48 - w^2}{4w}\right) = \frac{1}{4}(48w - w^3) \quad ✡$$

$$V = 12w - \frac{w^3}{4} \quad ✡$$

Optimize

Set

$$V' = 12 - \frac{3w^2}{4} = 0$$

C.N.

$$12 = \frac{3w^2}{4} \longrightarrow 12\left(\frac{4}{3}\right) = w^2$$

$$16 = w^2$$

plug C.N. into (✡)

Volume EQ for

max volume. $V = 12(4) - \frac{4^3}{4} \cdot \frac{64}{4} = 16$ $\quad w = 4$

$$= 48 - 16 = \boxed{32}$$

(51) objective : Area , $A = \frac{1}{2} r^2 \theta$

Constraint : Perimeter , $10 = 2r + r\theta$

$10 - 2r = r\theta \rightarrow \theta = \frac{10-2r}{r}$

Plug into Area : $A = \frac{1}{2} r^2 \left(\frac{10-2r}{r}\right)$

$= \frac{1}{2} r (10-2r) = 5r - r^2$

$A' = 5 - 2r = 0 \rightarrow 5 = 2r$

$r = \frac{5}{2}$

$A\left(\frac{5}{2}\right) = 5\left(\frac{5}{2}\right) - \left(\frac{5}{2}\right)^2 = 6.25$

(57) Decreasing & Concave down

$$f(x) = xe^{-x}$$

Need 1st Deriv. Test & Concavity test

$$f'(x) = x(-e^{-x}) + (1)e^{-x}$$

$$= e^{-x}(1-x) = 0 \quad \text{C.N. } x=1$$

inc. Dec.

$f'(x) > 0$ | $f'(x) < 0$

$x=1$

$x=0$ $x=2$

Dec. on $(1, \infty)$

$$f''(x) = e^{-x}(-1) + -e^{-x}(1-x)$$

$$= e^{-x}(-1 - 1 + x)$$

$$= e^{-x}(x-2) = 0 \quad \text{PIP } x=2$$

-- ++

$f''(x) < 0$ | $f''(x) > 0$

$x=2$

$x=0$. $x=3$

concave down on $(-\infty, 2)$

Concave down + Decreasing $(1, 2)$