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Lesson 10

Quotient Rule & Trig Derivatives

Last time Product rule

$$\frac{d}{dx}\left[h(x)\,g(x)\right] = h'(x)\,g(x) + h(x)\,g'(x)$$

Quotient Rule $g(x) \neq 0$

$$\frac{d}{dx}\left[\frac{h(x)}{g(x)}\right] = \frac{h'(x)\,g(x) - h(x)\,g'(x)}{(g(x))^2}$$

Rest of Trig derivatives

$$\frac{d}{dx}[\tan x] = \sec^2 x$$

$$\frac{d}{dx}[\sec x] = \sec x \tan x$$

$$\frac{d}{dx}[\cot x] = -\csc^2 x$$

$$\frac{d}{dx}[\csc x] = -\csc x \cot x$$

- Power rule
- $\frac{d}{dx}[\sin x] = \cos x$
- $\frac{d}{dx}[\cos x] = -\sin x$
- $\frac{d}{dx}[e^x] = e^x$

All together, these are the derivative rules you need on the exam

Example 1 Let $y = \dfrac{x^2+1}{3x^3+4}$ find y'

$h(x) = x^2+1$, $g(x) = 3x^3+4$

h' g h g'

$$y' = \frac{(2x)(3x^3+4) - (x^2+1)(9x^2)}{g^2 \longrightarrow (3x^3+4)^2}$$

$$= \frac{6x^4+6x-9x^4-9x^2}{(3x^3+4)^2} = \frac{-3x^4-9x^2+6x}{(3x^3+4)^2}$$

$$= \frac{-3x(x^3+9x-2)}{(3x^3+4)^2}$$

Example 2 Let $f(x) = \dfrac{\cos x}{5x-2}$ Find $f'(\pi)$

h' g h g'

$$f'(x) = \frac{(-\sin x)(5x-2) - (\cos x)(5)}{(5x-2)^2 \leftarrow g^2}$$

$$= \frac{-5x\sin x + 2\sin x - 5\cos x}{(5x-2)^2}$$

$$f'(\pi) = \frac{0+0-5(-1)}{(5(\pi)-2)^2} = \frac{5}{(5\pi-2)^2}$$

Example 3 Let $f(t) = \dfrac{e^t}{3-\sqrt{t}}$

Find $f'(1)$

$$f'(t) = \frac{\overbrace{e^t}^{h'}\overbrace{(3-\sqrt{t})}^{g} - \overbrace{e^t}^{h}\overbrace{(-\frac{1}{2}t^{-1/2})}^{g'}}{(3-\sqrt{t})^2}$$

$$= \frac{3e^t - e^t\sqrt{t} + \frac{1}{2}e^t t^{-1/2}}{(3-\sqrt{t})^2}$$ ← find to calc into lan. capac

$$= \frac{\frac{1}{2}e^t t^{-1/2}(6t^{1/2} - 2t + 1)}{(3-\sqrt{t})^2}$$

$$= \frac{e^t(6\sqrt{t} - 2t + 1)}{2\sqrt{t}(3-\sqrt{t})^2}$$ ← multiple choice could look like this

$$f'(1) = \frac{e(6-2+1)}{2(3-1)^2} = \frac{5e}{8}$$

Example 4 Given $y = 4\dfrac{(c^2 - x^2)}{c^2 + x^2}$ where c is a constant,

Find y'.

$$y' = \frac{\overset{h'}{4(-2x)}\overset{g}{(c^2+x^2)} - \overset{g'}{4(c^2-x^2)}(2x)}{(c^2+x^2)^2}$$

$$= \frac{4(-2xc^2 - \cancel{2x^3} - 2xc^2 + \cancel{2x^3})}{(c^2+x^2)^2}$$

$$= \frac{-16c^2x}{(c^2+x^2)^2}$$

Example 5 Let $f(x) = \dfrac{x}{4+x^2}$ find $f'(x)$

$$f'(x) = \frac{\overset{h'}{(1)}\overset{g}{(4+x^2)} - \overset{h}{x}\overset{g'}{(2x)}}{(4+x^2)^2}$$

$$= \frac{4+x^2-2x^2}{(4+x^2)^2} = \frac{4-x^2}{(4+x^2)^2}$$

Example 6 $y = 6x^3 \sec x$ find y'

(a)

$$y' = \overset{h'}{(18x^2)}\overset{g}{\sec x} + \overset{h}{(6x^3)}\overset{g'}{\sec x \tan x}$$

$$= 6x^2 \sec x (3 + \tan x)$$

(b) $y = 7e^x \cot x$ find y'

$$y' = \overset{h'}{7e^x}\overset{g}{(\cot x)} + \overset{h}{7e^x}(-\csc^2 x)$$

$$= 7e^x(\cot x - \csc^2 x)$$

(c) $$f(x) = \frac{\tan x}{1 + \csc x}$$

$$f'(x) = \frac{\overset{h'}{(\sec^2 x)}\overset{g}{(1+\csc x)} - (\tan x)(-\csc x \cot x)}{(1+\csc x)^2}$$

$$= \frac{\sec^2 x + \sec^2 \csc x + \tan x \csc x \cot x}{(1+\csc x)^2}$$