

02.03.22

Lesson 11

The chain Rule

Last time: Quotient Rule (on exam!)

$$\frac{d}{dx} [f(x)g(x)] = \frac{f'(x)g(x) - f(x)g'(x)}{g^2(x)}$$

Today Chain Rule

If $f(x)$ and $g(x)$ are differentiable then

$$\frac{d}{dx} [f(g(x))] = f'(g(x))g'(x)$$

Other format: $y = f(u)$, $u = g(x)$

then $y = f(g(x))$

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

(will use more in the future)

Example 1 Let $y = (x^3 + 2x)^2$

find y' .

Solution

$$f(x) = x^2, \quad g(x) = x^3 + 2x$$

$$f'(x) = 2x, \quad g'(x) = 3x^2 + 2$$

$$y' = f'(g(x)) \cdot g'(x)$$

$$= 2(x^3 + 2x)(3x^2 + 2)$$

|| Derive the
out side,
keep the
inside, derive
the inside ||

Example 2 Let $y = 3(7x^4 - 3x^2)^5$

$$y' = 3 \underbrace{(5(7x^4 - 3x^2)^4)}_{f'(g(x))} \underbrace{(7 \cdot 4x^3 - 3 \cdot 2x)}_{g'(x)}$$

$$= 15(7x^4 - 3x^2)^4(28x^3 - 6x)$$

Find $y'(1) = 15(7 - 3)^4(28 - 6)$

$$= 15(4)^4(22)$$

$$= 84480$$

Example 3 $f(x) = \sqrt[3]{7+4x^7}$

find $f'(x)$.

write $f(x) = h(g(x))$

$$h(x) = x^{1/3}$$

$$g(x) = 7+4x^7$$

$$h'(x) = \frac{1}{3} x^{-2/3}$$

$$g'(x) = 4 \cdot 7 x^6 \\ = 28 x^6$$

$$f'(x) = h'(g(x)) g'(x) \\ = \frac{1}{3} (7+4x^7)^{-2/3} 28 x^6 \\ = \frac{28}{3} \frac{x^6}{(7+4x^7)^{2/3}}$$

Example 4

$$y = \frac{1}{(1-2x+x^3)^3}$$

find y' .

$$y = (1-2x+x^3)^{-3}$$

$$y' = -3(1-2x+x^3)^{-4} (-2+3x^2)$$

$$= \frac{-3(-2+3x^2)}{(1-2x+x^3)^4}$$

Note! if no x in the numerator you can use chain rule instead of quotient

Example 5

$$g(x) = \left(\frac{4x}{3x^2+2} \right)^3$$

Find $g'(2)$,

$$g(x) = f(h(x))$$

$$f(x) = x^3$$

$$f'(x) = 3x^2$$

$$h(x) = \frac{4x}{3x^2+2}$$

$$h'(x) = \frac{4(3x^2+2) - 4x(6x)}{(3x^2+2)^2}$$

$$g'(x) = f'(h(x)) h'(x)$$

$$= 3 \left(\frac{4x}{3x^2+2} \right)^2 \left(\frac{8-12x^2}{(3x^2+2)^2} \right)$$

$$= 3 \left(\frac{16x^2(8-12x^2)}{(3x^2+2)^4} \right)$$

$$= \frac{12x^2 + 8 - 24x^2}{(3x^2+2)^2}$$

$$= \frac{8 - 12x^2}{(3x^2+2)^2}$$

Example 6 Let $y = 7(\cos x + 3e^x + 9)^3$

find $y'(0)$

$$y' = 7 \cdot 3 \overbrace{(\cos x + 3e^x + 9)^2}^{g(x)} \overbrace{(-\sin x + 3e^x)}^{g'(x)}$$

$$= 21(\cos x + 3e^x + 9)^2 (3e^x - \sin x)$$

$$y'(0) = 21(1 + 3 + 9)^2 (3 - 0)$$

$$= 21(13)^2 \cdot 3 = 10647$$