02.14.22

Lesson 13 Higher Order Derivatives

We've been taking derivatives of functions for the last few weeks, but there's no reason we can't take the derivative again and again.

Example 1 Let $f(x) = 4\sqrt{x} - \frac{5}{x}$

Find $f''(x)$, the second derivative and $f''(4)$

$$f(x) = 4x^{1/2} - 5x^{-1}$$

$$f'(x) = 4 \cdot \frac{1}{2} x^{-1/2} - 5(-x^{-2})$$

$$= 2x^{-1/2} + 5x^{-2}$$

$$f''(x) = 2\left(-\frac{1}{2}x^{-3/2}\right) + 5(-2x^{-3})$$

$$= \frac{-1}{x^{3/2}} - \frac{10}{x^3}$$

$$f''(4) = \frac{-1}{4^{3/2}} - \frac{10}{4^3} = -\frac{1}{8} - \frac{10}{64} = \frac{-18}{64} = \frac{-9}{32}$$

Notation

$f'(x)$ $\quad \frac{dy}{dx}$ $\qquad f^{(4)}(x), f^{(5)}(x)$, ect.

$f''(x)$ $\quad \frac{d^2y}{dx^2}$

$f'''(x)$ $\quad \frac{d^3y}{dx^3}$

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Example 2

Find the second derivative of

$$g(x) = \underbrace{7e^{3x}}_{f}\ \underbrace{\sin(4x)}_{h}$$

$$g'(x) = \underbrace{7(3e^{3x})}_{f'}\ \underbrace{\sin(4x)}_{h} + \underbrace{7e^{3x}}_{f}\ \underbrace{\cos(4x)\,4}_{h'}$$ (Product Rule)

$$= 7e^{3x}(3\sin(4x) + 4\cos(4x))$$ (Product Rule again)

$$g''(x) = 7(3e^{3x})(3\sin(4x) + 4\cos(4x)) + 7e^{3x}(3\cos(4x)4 + 4(-\sin(4x)\cdot 4))$$

$$= 63e^{3x}\sin(4x) - 112e^{3x}\sin(4x) + 84e^{3x}\cos(4x) + 84e^{3x}\cos(4x)$$

$$= 168e^{3x}\cos(4x) - 49e^{3x}\sin(4x)$$

Interpretation of Higher order derivatives

$s(t)$ position

$s'(t) = v(t)$ velocity

$s''(t) = v'(t) = \underline{a(t)}$ acceleration m/s^2

Example 3 A particle is traveling on a straight line with a position function of

$$s(t) = \frac{3t^2+1}{4t+3}$$

What is the particle's acceleration when $t=2$?

$$v(t) = \frac{(3\cdot 2t)(4t+3) - (3t^2+1)(4)}{(4t+3)^2} = \frac{6t(4t+3) - 4(3t^2+1)}{(4t+3)^2}$$

$$= \frac{24t^2 + 18t - 12t^2 - 4}{(4t+3)^2}$$

$$= \frac{12t^2 + 18t - 4}{(4t+3)^2}$$

$$a(t) = \frac{(24t+18)(4t+3)^2 - (12t^2+18t-4)\,2(4t+3)\,4}{(4t+3)^4}$$

$$= \frac{\cancel{(4t+3)}\left((24t+18)(4t+3) - 8(12t^2+18t-4)\right)}{(4t+3)^{\cancel{4}\,3}}$$

$$a(2) = \frac{(24(2)+18)(4(2)+3) - 8(12(2)^2+18(2)-4)}{(4(2)+3)^3}$$

$$= 0.0646$$

Example 4 Find the fifth derivative of $f(x)$

if $f^{(4)}(x) = 3\sec(4-5x)$

$$f^{(5)}(x) = 3\sec(4-5x)\tan(4-5x)(-5)$$

$$= -15\sec(4-5x)\tan(4-5x)$$

Example 5 Find the 4th derivative of

$$f(x) = e^{-2x}$$

$$f'(x) = -2e^{-2x}$$

$$f''(x) = 4e^{-2x}$$

$$f'''(x) = -8e^{-2x}$$

$$f^{(4)}(x) = 16e^{-2x}$$

Example 6   Find the second derivative of

$$h(x) = 11x^3 \ln(2x)$$

$$h'(x) = 33x^2 \ln(2x) + 11x^3 \frac{2}{2x}$$

$$= 33x^2 \ln(2x) + 11x^2$$

$$h'(x) = 33(2x)\ln(2x) + 33x^2 \frac{2}{2x} + 22x$$

$$= 66x \ln(2x) + 33x + 22x$$

$$= 66x \ln(2x) + 55x$$

$$= 11x(6\ln(2x) + 5)$$