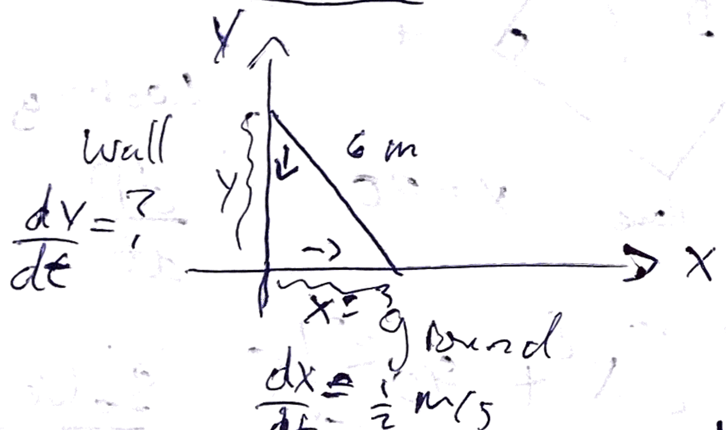
Lesson 16

Related Rates Part 2

Example 1

A ladder 6 meters long rests on horizontal ground and leans against a vertical wall. The foot of the ladder is pulled away from the wall at the rate of 0.5 m/sec. How fast is the top sliding down the wall when the foot of the ladder is 3 m from the wall? (Do not need "-" in the answer.)



x = 3

Eq

$$x^2 + y^2 = 36$$

$$\frac{d}{dt}[x^2 + y^2] = \frac{d}{dt}[36]$$

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$$

$$\frac{dy}{dt} = -\frac{x}{y}\frac{dx}{dt}$$

Solve for y

$$3^2 + y^2 = 36$$

$$y = \sqrt{36 - 9}$$

$$= \cancel{\sqrt{28}}\ \sqrt{27}$$

Plug in info

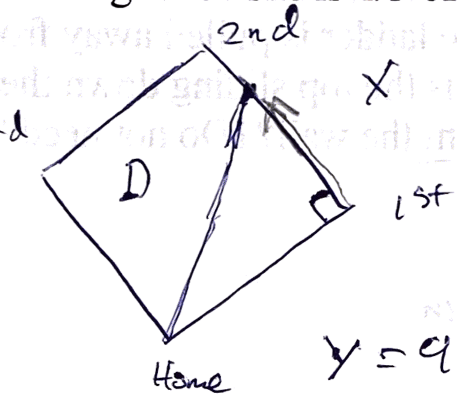
$$\frac{-3}{\sqrt{27}}\left(\frac{1}{2}\right) \text{ m/s}$$

$$= \frac{-3}{2\sqrt{27}} \text{ m/s}$$

The ladder is sliding down the wall at a rate of $\frac{3}{2\sqrt{27}}$ m/s when x = 3 m.

Example 2

A softball diamond is a square 90 ft on a side. A player runs from first base to second base at 12 ft/sec. At what rate is the player's distance from home base increasing when she is 2/3 of the way from first to second base?



$x = \frac{2}{3}(90) = 60$

$\frac{dx}{dt} = 12$

$D^2 = x^2 + y^2$

$y = 90$

Looking for

$\frac{dD}{dt}$

$$\frac{d}{dt}[D^2] = \frac{d}{dt}[x^2 + 90^2]$$

$$2D\frac{dD}{dt} = 2x\frac{dx}{dt} + 0$$

$$\frac{dD}{dt} = \frac{x}{D}\frac{dx}{dt}$$

Solve for D

$D = \sqrt{60^2 + 90^2}$

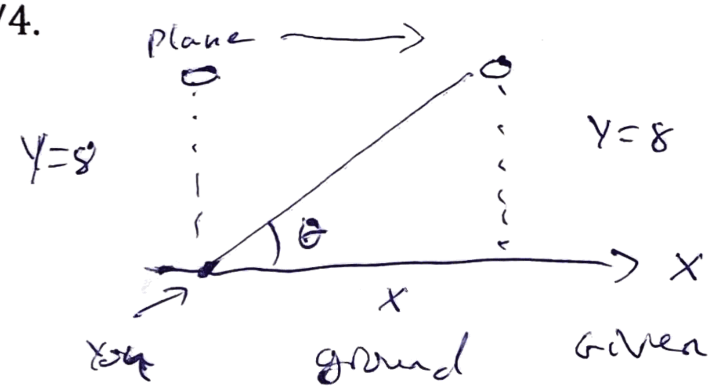
$= \sqrt{11700}$

Plug in info

$$\frac{dD}{dt} = \frac{60}{\sqrt{11700}} \cdot 12 \text{ ft/sec} = \frac{720}{\sqrt{11700}} \text{ ft/sec}$$

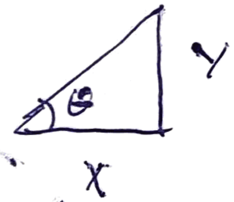
Example 4

A plane is flying away from you at a speed of 250 mph at an altitude of 8 miles. Find the rate at which the angle of elevation is decreasing when the angle is $\pi/4$.



Given $\frac{dx}{dt} = 250$ m/h

want $\frac{d\theta}{dt} = ?$



$$\tan\theta = \frac{Y}{X} = \frac{8}{X}$$

$$\frac{d}{dt}(\tan\theta) = \frac{d}{dt}[8x^{-1}]$$

$$\sec^2\theta \frac{d\theta}{dt} = -8x^{-2}\frac{dx}{dt}$$

$$\frac{d\theta}{dt} = \frac{-8x^{-2}}{\sec^2\theta}\frac{dx}{dt}$$

$$= \frac{-8\cos^2\theta}{x^2}\frac{dx}{dt}$$

$$\theta = \frac{\pi}{4}$$

$$\tan\frac{\pi}{4} = \frac{8}{X}$$

$$1 = \frac{8}{X}\ ;\ x = 8$$

$$\cos\frac{\pi}{4} = \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}}$$

plug in into

$$\frac{d\theta}{dt} = \frac{-8\left(\frac{1}{\sqrt{2}}\right)^2(250)}{8^2} = -\frac{250}{2\cdot 8} = \frac{-250}{16} \text{ rad/h}$$

$$\frac{-250}{16} = \frac{-125}{8}$$

Quiz 4 (L. 9-12)

Solutions

Problem 1 Find $f'(1)$

$$f(x) = \underbrace{e^{2x}}_{h(x)} \underbrace{(2x-1)^3}_{g(x)}$$

$h(x) = e^{2x}$

$h'(x) = 2e^{2x}$

(chain rule)

$g(x) = (2x-1)^3$

$g'(x) = 3(2x-1)^2(2)$ (chain rule)

$f'(x) = h'(x)\,g(x) + h(x)\,g'(x)$ (Product rule)

$= 2e^{2x}(2x-1)^3 + e^{2x}\,6\,(2x-1)^2$

$f'(1) = 2e^{2(1)}(2(1)-1)^3 + e^{2(1)}\,6\,(2(1)-1)^2$

$= \boxed{8e^2}$

Problem 2 Find $f'(0)$; $f(x) = \dfrac{\sin(2x)}{(1+3x)^2}$ $\dfrac{g(x)}{h(x)}$

$g(x) = \sin(2x)$

$g'(x) = \cos(2x)(2)$

(Chain rule)

$h(x) = (1+3x)^2$

$h'(x) = 2(1+3x)(3)$

$f'(x) = \dfrac{g'h - gh'}{g^2}$ (Quotient rule)

$$f'(x) = \frac{2\cos(2x)(1+3x)^2 - \sin(2x)(6(1+3x))}{(1+3x)^4}$$

$$f'(0) = \frac{2\underbrace{\cos(0)}_{=1}(1) - \underbrace{\sin(0)}_{=0}(6)}{(1+3(0))^4} = \frac{2}{1} = \boxed{2}$$