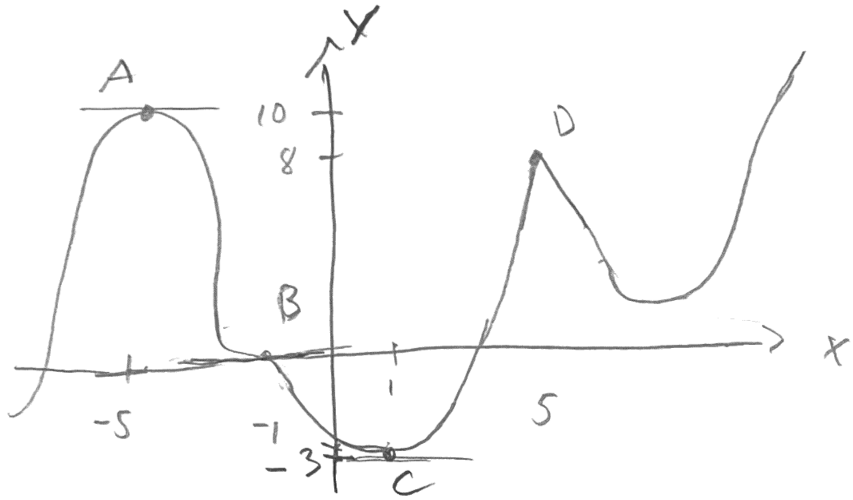
02.23.22 Lesson 17

Relative Extrema and Critical points

Idea of Relative Extrema:



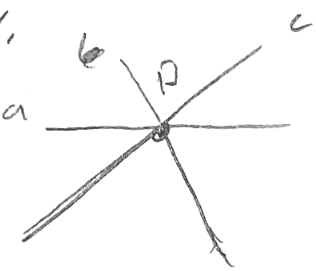
- We call A and D <u>relative maximums</u> because they are higher than the points close by.
- Similarly, we refer to C as a relative minimum.

- If we don't have a graph, we can find possible extrema or <u>critical points</u> by finding where the slope of the tangent line is zero. That is, where $f'(x) = 0$, or <u>where $f'(x)$ DNE</u> but $f(x)$ Does exist.
- Notice Point B. $f'(B) = 0$ but it has neighbors that are both higher and lower than it. Hence, B is <u>not</u> a rel. max or rel. min.

In Lesson 18 we will learn the <u>First Derivative Test</u> which lets us determine which critical points are rel. max, rel. min or neither.

- Today we will just practice finding C. P.

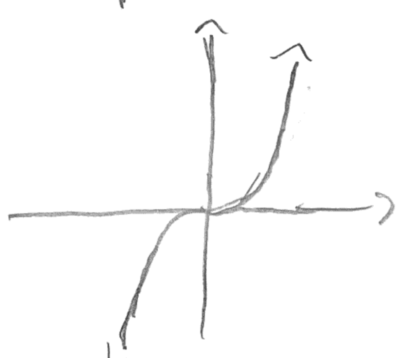
- Finally, Notice point D. Point D is a sharp corner and the derivative does not exist. Why? Well, there is more than one tangent line at D! Therefore, the derivative cannot be defined uniquely.



Three possible tangent lines a, b, c.

$\boxed{\text{This is why } x \text{ where } f'(x) \text{ DNE is also a critical point}}$

- To get an idea of point B think of $f(x) = x^3$



$f'(x) = 3x^2 = 0$

at $x=0$ but from the graph we see $x=0$ is not a rel. max or rel. min.

Example 1

Find the critical numbers

$$Y = 3 + 6x - \frac{5x^3}{3}$$

Set $y' = 0$, solve for x.

$$y' = 6 - 5x^2 = 0 \quad \Rightarrow \quad 5x^2 = 6$$

$$\Rightarrow \quad x^2 = \frac{6}{5}$$

$$\Rightarrow \quad x = \pm\sqrt{\frac{6}{5}}$$

critical numbers at $x = -\sqrt{\frac{6}{5}}$ and $x = \sqrt{\frac{6}{5}}$

Example 2 (No real roots)

Find the critical numbers of

$$f(x) = x^3 + 4x^2 + 6x$$

Set $f'(x) = 0$, solve for x

$$f'(x) = 3x^2 + 8x + 6$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{8 \pm \sqrt{8^2 - 4(3)(6)}}{2(3)}$$

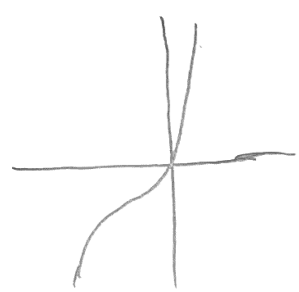
$$= \frac{8 \pm \sqrt{64 - 72}}{6}$$

$\sqrt{-8}$

↑ not a real number

So no real roots.

$\boxed{\text{no critical numbers}}$



← graph

Example 3 Find the critical numbers

$$y = 2x^3 - \frac{3}{x^2}$$

$$y' = 6x^2 - 3(-2x^{-3}) = 0$$

$$y' = 6x^2 + \frac{6}{x^3} = 0$$

$$6x^2\left(1 + \frac{1}{x^5}\right) = 0$$

- y' DNE when $x=0$ but neither does y, so $x=0$ is not a critical number.

- $1 + \frac{1}{x^5} = 0 \Rightarrow 1 = -\frac{1}{x^5} \Rightarrow x^5 = -1 \Rightarrow x = -1$

- $y' = 0$ when $x=-1$, so $\boxed{x=-1 \text{ is a Critical Number}}$

Example 4 Find the critical numbers

$$y = \frac{2x^2+3}{3x}$$

$$y' = \frac{3x(4x) - (2x^2+3)(3)}{(3x)^2} = \frac{12x^2 - 6x^2 - 9}{9x^2}$$

$$= \frac{6x^2-9}{9x^2} = \frac{3(x^2-3)}{9x^2}$$

critical numbers

$x = \pm\sqrt{3}$ $\leftarrow$ where $f'(x) = 0$

$$= \frac{x^2-3}{3x^2} = 0$$

$$x^2 = 3$$

Example 4 ...continued

Check where $f'(x)$ DNE.

$f'(x)$ DNE when $x=0$ but $x=0$ is not in the domain of $f(x)$ because $f(0)$ DNE. Therefore, $x=0$ is not a critical point

Example 5 Find the critical numbers for

$$y=3x^5e^{2x-1}$$

$$y'=15x^4e^{2x-1}+3x^5e^{2x-1}(2)=0$$

$$3x^4e^{2x-1}(5+2x)=0$$

$$x=0 \quad \text{and} \quad x=-\frac{5}{2}$$

<u>Example 6</u> Find the critical numbers.

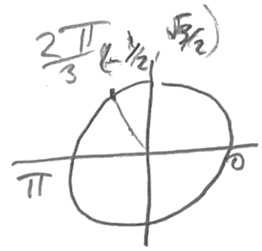
$$y = 2\sin(4x) + 4x$$

on the interval of $(0, \pi]$

$$y' = \underbrace{2\cos(4x)\,4}_{\text{(chain rule)}} + 4 = 0$$

$$8\cos(4x) = -4$$

$$\cos(4x) = -\frac{1}{2}$$



only happens when $4x = \frac{2\pi}{3}$

so $x = \frac{2\pi}{4 \cdot 3} = \boxed{\frac{\pi}{6}}$ critical number