1.12.22

Lesson 2

Finding limits and One-sided limits numerically

Idea: A limit describes what happens to the output of a function close to a value

To understand "close to" lets fill out this table for f(x) from the warm up

Example 1

$$f(x) = \frac{x^2 + x - 6}{x^2 - x - 2} = \frac{(x+3)(x-2)}{(x+1)(x-2)}$$

Find $\lim_{x \to 2} f(x)$

both are close to 2

x	1.9	1.99	1.999	2	2.001	2.01	2.1
f(x)	1.689	1.6688	1.666	–	1.666	1.6644	1.645

So $\lim_{x \to 2} f(x) = 1.666 \approx \frac{5}{3}$ which is what we expected.

So limits let us understand "holes" and what happens around other domain issues like vertical asymptotes,

- For values in the domain of a function, the limit and the function can agree.

Example 2 Find $\lim_{x \to 1} (3x - 1)$

x	0.9	0.99	0.999	1	1.001	1.01	1.1
f(x)	1.7	1.97	1.997	—	2.003	2.03	2.3

So $\lim_{x \to 1} 3x-1 = 2 = f(1)$

Example 3 Let $f(x) = \frac{3}{(x-3)^2}$, find $\lim_{x \to 3} f(x)$

- What do you expect?

x	2.9	2.99	2.999	3	3.001	3.01	3.1
f(x)	300	30000	3,000,000	—	3,000,000	30000	300

So $\lim_{x \to 3} f(x) = \infty$

Recall f(x) has a vertical asymptote at $x = 3$

- So far close meant from both above and below (smaller and larger) our chosen value, But it doesn't have to.

- <u>One-sided-limits</u>

$\lim_{x \to 3^-} f(x)$ - left-sided limit, choose values on the left (smaller) side of 3

$\lim_{x \to 3^+} f(x)$ - Right-sided limit, choose values on the right (larger) side of 3.

-(Look at left an right limits from previous examples)

- Notice so far that the left and right limits in our examples have always matched up

- This is not always the case.

<u>Example 4</u> let $f(x) = \dfrac{2}{3x+6}$, find $\lim_{x \to -2^-} f(x)$

$\lim_{x \to -2^+} f(x)$, and $\lim_{x \to -2} f(x)$

Example 4 ... continued

left →

x	-2.1	-2.01	-2.001	-2
$f(x)$	-6.6666	-66.666	-666.666	$-$

$$\lim_{x \to -2^-} f(x) = -\infty$$

Right →

x	-2	-1.999	-1.99	-1.9
$f(x)$	$-$	666.666	66.666	6.666

$$\lim_{x \to -2^+} f(x) = +\infty$$

$$\lim_{x \to -2} f(x) \quad \text{DNE}$$

Example 5

Let $f(x) = \dfrac{-1}{x+1}$ find

$\lim_{x \to -1^-} f(x)$, $\lim_{x \to -1^+} f(x)$, $\lim_{x \to -1} f(x)$

x	-1.1	-1.01	-1.001	-1	$-.999$	$-.99$	$-.9$
$f(x)$	10	100	1000	$-$	-1000	-100	-10

$\lim_{x \to -1^-} f(x) = \infty$, $\lim_{x \to -1^+} f(x) = -\infty$, $\lim_{x \to -1} f(x)$ DNE

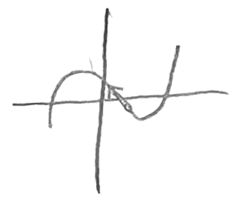
$$f(-1.1) = \frac{-1}{-1.1+1} = \frac{-1}{-.1} = \frac{1}{\frac{1}{10}} = 10$$

Formal Definitions

Left-sided limit if f(x) approaches L as x approaches c from the left we say, the limit of f(x) as x approaches c from the left is L

$$\lim_{x \to c^-} f(x) = L$$

Right-sided limit if f(x) approaches L as x approaches c from the right we say, the limit of f(x) as x approaches c from the right is L



$$\lim_{x \to c^+} f(x) = L$$

Limit if f(x) approaches L as x approaches c, we say the limit of f(x) as x approaches c is L.

$$\lim_{x \to c} f(x) = L$$

- If f(x) increases or decreases without bound as x approaches c we write

(from the left) $\lim_{x \to c^-} f(x) = \infty$ or $\lim_{x \to c^-} f(x) = -\infty$

(from the right) $\lim_{x \to c^+} f(x) = \infty$ or $\lim_{x \to c^+} f(x) = -\infty$

(General limit) $\lim_{x \to c} f(x) = \infty$ or $\lim_{x \to c} f(x) = -\infty$

Theorem $\lim_{x \to c} f(x)$ exists if and only if

$$\lim_{x \to c^-} f(x) = \lim_{x \to c^+} f(x)$$

In this case all three are equal.

- For the limit to exist the function must approach the same value from the left and right.

Example 5 Let $f(x) = \frac{\sin x}{x}$, find $\lim_{x \to 0} f(x)$

x	-.1	-.01	-.001	0	.001	.01	.1
f(x)	.9983	.9999	.99999	–	.99999	.9999	.9983

So $\lim_{x \to 0} f(x) = 1$

Notice this behaves like the "hole" example but there is nothing easy to cancles out.

Extra $f(x) = \frac{1}{(x-5)^3}$, $g(x) = -\frac{1}{(x+5)^2}$