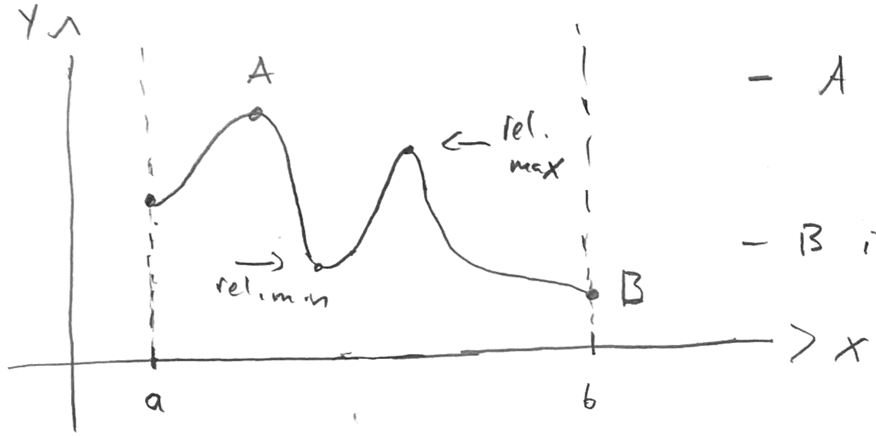
03.04.22

Lesson 20

Absolute Extrema



- A is absolute maximum.
- B is absolute minimum.

Idea If we restrict our search to a closed interval $[a, b]$, the absolute maximum is the point $x = c$ in $[a, b]$ where $f(c)$ is larger than every other value of $f(x)$

the absolute minimum is the point $x = c$ in $[a, b]$ where $f(c)$ is smaller than every other value of $f(x)$.

- why is this different from relative maximum and minimum?

(1) We have to consider end points as well as critical numbers.

(2) We have to compare $f(c)$ with each other.

Notation / Definition

- closed interval: $[a,b]$

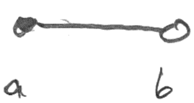
includes end points

- open interval: (a,b)

does not include end points

- half-open / half-closed interval

$[a,b)$



or

$(a,b]$

Theorem: If $f(x)$ is continuous on a closed interval $[a,b]$, then $f(x)$ has both an absolute maximum and an absolute minimum on the interval.

The absolute extrema only occure at

1. Critical numbers <u>or</u>
2. End points

Example 1 Find the absolute extrema of

$$y = 4x^2 - 3x + 11$$

on the closed interval $[-1, 2]$

Solution

(1) Find C.N. and end points

(2) Compare the y values at these points.

$y' = 8x - 3 = 0$ $\qquad x = \frac{3}{8}$ C.N.

$x = -1, \quad x = 2$ End Points

$$y\left(\tfrac{3}{8}\right) = 4\left(\tfrac{3}{8}\right)^2 - 3\left(\tfrac{3}{8}\right) + 11 = \frac{668}{64} \approx 10.43$$

$$y(-1) = 4 + 3 + 11 = 18$$

$$y(2) = 4(2)^2 - 3(2) + 11 = 16 - 6 + 11 = 21$$

absolute maximum at $(2, 21)$

absolute minimum at $\left(\frac{3}{8}, \frac{668}{64}\right)$

Example 2 Find the absolute extrema of

$$f(x) = 5x^4 - 12x^3 + 6$$

on the closed interval $[-1, 2]$

$$f'(x) = 20x^3 - 36x^2 = 0$$

$$4x^2(5x - 9) = 0$$

C.N. $x = 0$ $x = \frac{9}{5}$

End Points: $x = -1$ and $x = 2$

$f(0) = 6$

$f\left(\frac{9}{5}\right) = -11.496$

$f(-1) = 5 + 12 + 6 = 23$

$f(2) = 5(2)^4 - 12(2)^3 + 6 = -10$

absolute maximum at

$(-1, 23)$

absolute minimum at

$\left(\frac{9}{5}, -11.596\right)$

Example 3   Find the absolute extrema of

$f(x) = 3xe^{-x} + 7$ on the closed interval $[0,4]$

$$f'(x) = 3e^{-x} + 3x(-e^{-x})$$
$$= 3e^{-x}(1-x) = 0$$

C.N. at $x=1$, end points $x=0$, $x=4$

$$f(0) = 3(0)e^{-(0)} + 7 = 7$$

$$f(1) = 3(1)e^{-(1)} + 7 = \frac{3}{e} + 7 \approx 8.104$$

$$f(4) = 3(4)e^{-(4)} + 7 = \frac{12}{e^4} + 7 \approx 11.41$$

absolute minimum at $(0,7)$

absolute maximum at $\left(4, \frac{12}{e^4} + 7\right)$

Note: Sometimes you are asked to find absolute extrema on open or half-open intervals, In those cases there will only be one critical numbers.

Example 4: Find the absolute minimum of $y = \frac{3x^2}{x+2}$ on the interval $(-3, 4]$

$$y' = \frac{(x+2)\,6x - 3x^2(1)}{(x+2)^2}$$

$$= \frac{6x^2 + 12x - 3x^2}{(x+2)^2} = \frac{3x^2+12x}{(x+2)^2} = \frac{3x(x+4)}{(x+2)^2} = 0$$

Note: $x=-2$ is not a C.N. b/c $y'(-2)$ DNE but $y(-2)$ DNE as well.

C.N. at $x=0$ and $x=-4$

continues...

Example 4 ... continued

Note $x = -4$ is not in $(-3, 4]$ so don't consider it.

Note $x = -1$ is not in $(-3, 4]$ so we don't consider it for an extrema.

Just need to check $x = 0$, C.N. and $x = 4$, end point.

$$f(0) = 0$$

$$f(4) = \frac{3(4)^2}{4+2} = \frac{48}{6} = 8$$

So $(0, 0)$ is the absolute minimum.