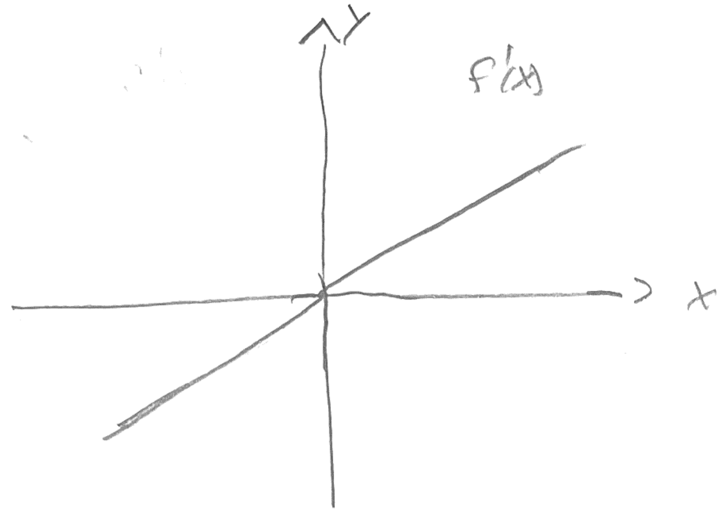
03.07.22

Graphical Interpretation of Derivatives

Today we will answer questions about the characteristics of a function based on the graph of its derivative



critical numbers: $x = 0$ $\quad$ ($f'(x) = 0$ or $f'(x)$ DNE)

Increasing interval: $(0, \infty)$ $\quad$ (where $f'(x) > 0$)

Decreasing interval: $(-\infty, 0)$ $\quad$ (where $f'(x) < 0$)

relative max : none $\quad$ ($f'(x) > 0$; $f'(x) < 0$)

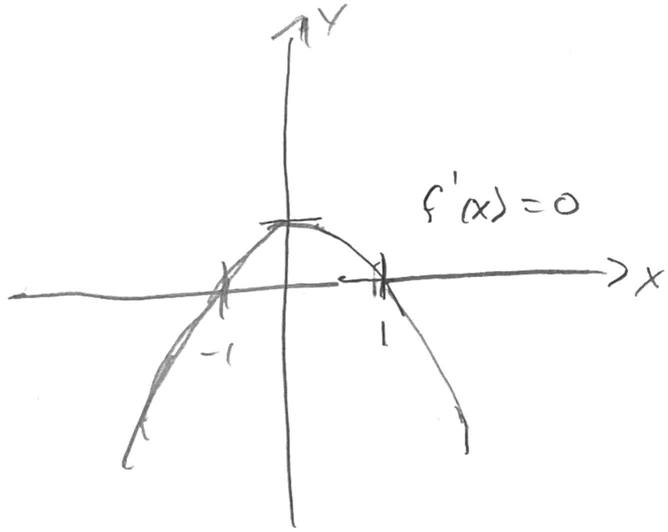
relative min : $x = 0$ $\quad$ ($f'(x) < 0$; $f'(x) > 0$

Concave up intervals: $(-\infty, \infty)$ $\quad$ (slope of $f'(x)$ is pos. everywhere)

Concave down intervals: none $\quad$ (slope of $f'(x)$ never neg.)

Inflection point: none $\quad$ (concavity does not change

Example 2



Critical Numbers: $x = -1$, $x = 1$ (where $f'(x) = 0$ or $f'(x)$ DNE)

Increasing Intervals: $(-1, 1)$ $(f'(x) > 0)$

Decreasing Intervals: $(-\infty, -1)$ and $(1, \infty)$ $f'(x) < 0$

Relative Max: $x = 1$ $f'(x) > 0$ $f'(x) < 0$

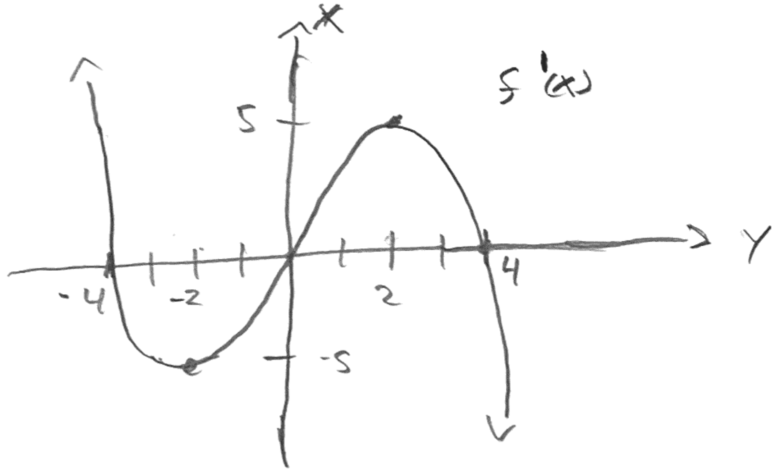
Relative Min: $x = -1$ $f'(x) < 0$ $f'(x) > 0$

Concave up: $(-\infty, 0)$: (slope of $f'(x)$ pos.)

Concave down: $(0, \infty)$: (slope of $f'(x)$ neg.)

Inflection points: $x = 0$ (concavity changes)

Example 3



Critical Numbers : $X=-4$, $X=0$, $X=4$ ($f'(x)=0$ or DNE)

Increasing interval: $(-\infty,-4)$, $(0,4)$ ($f'(x)>0$)

Decreasing interval: $(-4,0)$, $(4,\infty)$ ($f'(x)<0$)

Relative max : $X=-4$ $\quad f'(x)>0 \quad f'(x)<0$

$X=4$

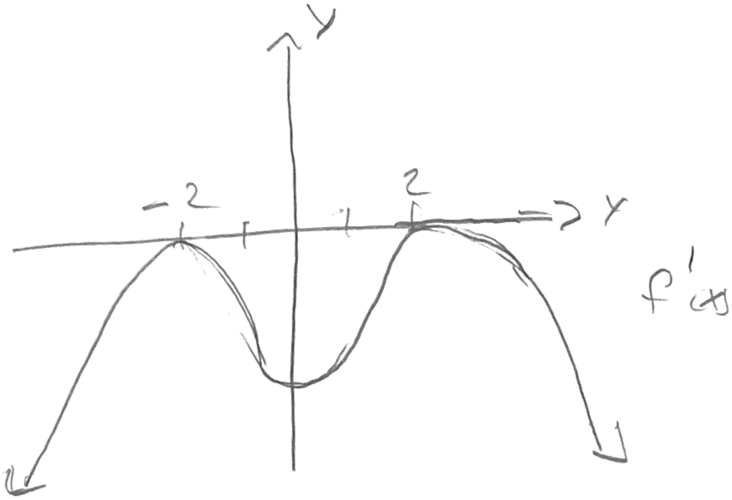
Relative min : $X=0$ $\quad f'(x)<0 \quad f'(x)>0$

Concave up : $(-2, 2)$ Slope of $f''(x)$ pos

concave down: $(-\infty,-2)$, $(2,\infty)$ Slope of $f'(x)$ neg

Inflection point: $X=-2$ and $x=2$ (concavity changes)

Example 4



critical numbers: $x = -2$, $x = 2$ ($f'(x) = 0$ or $f'(x)$ DNE)

Increasing interval: none ($f'(x) > 0$)

Decreasing interval: $(-\infty, \infty)$ ($f'(x) < 0$

Relative max: none, f is decreasing everywhere

Relative min: none, f is decreasing everywhere

concave up: $(-\infty, -2)$, $(0, 2)$ slope of $f'(x)$ is positive

concave down: $(-2, 0)$, $(2, \infty)$ slope of $f'(x)$ is negative

inflection points: $x = -2$, $x = 0$, $x = 2$ (concavity changes