Review Problems
Lessons 24-36 Solutions

(57)

$$\Delta x = \frac{4-1}{3} = 1 \quad , \quad x_i = 1 + i$$

$$x_0 = 1, \quad x_1 = 2, \quad x_2 = 3, \quad x_3 = 4$$

$$T_3 = \frac{1}{2} \underbrace{(1)}_{\Delta x} \left(e^{1^2-1} + 2e^{2^2-1} + 2e^{3^2-1} + e^{4^2-1} \right)$$

$$= \frac{1}{2} \left(1 + 2e^3 + 2e^8 + e^{15} \right) \qquad \text{(F)}$$

(55)

$$R_{100} - L_{100} = \sum_{n=1}^{100} f(x_i)\Delta x - \sum_{n=0}^{99} f(x_i)\Delta x$$

$$= \Delta x \left(f(x_{100}) - f(x_0) \right)$$

$$\left(\Delta x = \frac{10-0}{100} = \frac{1}{10} \qquad x_i = 0 + \frac{i}{10} \qquad f(x) = 7x \right)$$

$$= \frac{1}{10} \left(f(10) - f(0) \right) = \frac{1}{10}(70) = 7 \qquad \text{(F)}$$

(44)

$$S_A = \int v_A(t)\,dt = \int (t+2)\,dt = \frac{t^2}{2} + 2t + C$$

$$\underbrace{S_B}_{\text{Displacement}} = \int v_B(t)\,dt = \int 2t\,dt = t^2 + C$$

$$S_A(t) = S_B(t) \longrightarrow \frac{t^2}{2} + 2t = t^2$$

$$\frac{t^2}{2} - 2t = 0$$

$$t = 4 \quad \text{(D)} \qquad t\left(\frac{t}{2} - 2\right) = 0$$

(43) $\frac{dy}{dt} = -4y$ and $Y(3) = 42$, find $Y(5)$

$Y(t) = C e^{-4t}$

$Y(3) = C e^{-12} = 42 \rightarrow \quad C = 42 e^{12}$

$Y(t) = 4e^{12} e^{-4t}$

$Y(5) = 4 e^{12} e^{-4(5)} = 4 e^{12-20} = 4e^{-8}$ (D)

(42) $\int_0^3 a(t)\,dt = \int_0^3 -(t-3)^2\,dt = \int_0^3 -(t^2 - 6t + 9)\,dt$

$= \left(-\frac{t^3}{3} + 3t^2 - 9t\right)\Big|_0^3$

$= -\frac{27}{3} + 3(3)^2 - 9(3)$

$= -9 + 27 - 27$

$= -9$

The car's speed decreased by 9 mph 3 seconds after the breaks were applied. The initial speed was 60 mph so the New Speed is $60 - 9 = 51$ mph (D)

(24)

$$\int_0^8 f(x)dx - \int_1^8 f(x)dx = \int_0^1 f(x)\,dx$$

$$10 \quad - \quad 3 \quad = \quad 7$$

$$\int_1^4 f(x)dx = \int_0^4 f(x)\,dx - \int_0^1 f(x)dx = -7-7$$

$$= -14 \quad \text{(A)}$$

(2)

$$\Delta x = \frac{3-1}{4} = \frac{1}{2}, \qquad x_i = 1 + i\underbrace{\tfrac{1}{2}}_{\Delta x}$$

$$L_4 = \sum_{n=0}^{3} f(x_i)\Delta x = \sum_{n=0}^{3} \left(1+\tfrac{i}{2}\right)^3 dx$$

$$= \left(1^3 + \left(\tfrac{3}{2}\right)^3 + (2)^3 + \left(\tfrac{5}{2}\right)^3\right)\tfrac{1}{2}$$

$$= \left(1 + \frac{27}{8} + 8 + \frac{125}{8}\right)\frac{1}{2}$$

$$= \left(9 + \frac{152}{8}\right)\frac{1}{2} = (9+19)\frac{1}{2} = 14 \quad \text{(E)}$$

(20)

half-life 50000

$$y(t) = 2500\,e^{kt}$$

$$y(50000) = 2500\,e^{k(50000)} = \frac{1}{2}(2500)$$

$$k(50000) = \ln\left(\tfrac{1}{2}\right)$$

$$k = \frac{\ln\left(\frac{1}{2}\right)}{50000}$$

$$y(t) = 2500\,e^{\frac{\ln(\frac{1}{2})}{50000}t} \;; \quad y(65000) = 2500\,e^{\frac{\ln(\frac{1}{2})}{50000}(65000)}$$

$$= 1015 \text{ grams} \quad \text{(E)}$$

(37) $$\int_{\pi/6}^{\pi/3} (2\sec^2\theta + 3\theta)\, d\theta = 2\tan\theta + \frac{3}{2}\theta^2 \Big|_{\pi/6}^{\pi/3}$$

$$= \left(2\tan\left(\frac{\pi}{3}\right) + \frac{3}{2}\left(\frac{\pi}{3}\right)^2\right) - \left(2\tan\left(\frac{\pi}{6}\right) + \frac{3}{2}\left(\frac{\pi}{6}\right)^2\right)$$

$$= \left(2(\sqrt{3}) + \frac{\pi^2}{6}\right) - \left(\frac{2}{\sqrt{3}} + \frac{\pi^2}{24}\right)$$

$$= 2\sqrt{3} + \frac{4\pi^2}{24} - \frac{2}{\sqrt{3}} - \frac{\pi^2}{24}$$

$$= \frac{6\sqrt{3} - 2\sqrt{3}}{3} + \frac{\pi^2}{8} = \frac{4\sqrt{3}}{3} + \frac{\pi^2}{8} \quad \text{(C)}$$

(33) $$A(t) = 50000\, e^{.08t} = 200000$$

$$e^{.08t} = \frac{200000}{50000} = 4$$

$$\ln(e^{.08t}) = \ln 4$$

$$.08t = \ln 4 \rightarrow t = \frac{\ln 4}{.08} = 17.33 \quad \text{(D)}$$

Years

(39) $$\int_0^{16} \frac{x + \sqrt[4]{x}}{\sqrt{x}}\, dx = \int_0^{16} \left(x^{1/2} + x^{-1/4}\right) dx$$

$$= \left(\frac{2}{3}x^{3/2} + \frac{4}{3}x^{3/4}\right)\Big|_0^{16}$$

$$= \frac{2}{3}(16)^{3/2} + \frac{4}{3}(16)^{3/4}$$

$$= \frac{2}{3}(64) + \frac{4}{3}(8)$$

$$= \frac{128 + 32}{3} = \frac{160}{3} = \text{(A)}$$

(14) $\int_{-\pi}^{1}(3x+\pi)\,dx = \frac{3x^2}{2} + \pi x \Big|_{-\pi}^{1}$

$= \left(\frac{3}{2}(1)^2 + \pi(1)\right) - \left(\frac{3}{2}\pi^2 - \pi^2\right.$

$= \frac{3}{2} + \pi - \frac{\pi^2}{2}$ (A)