1.14.22

Finding Limits Graphically

Last time $\lim_{x \to c} f(x)$ exists if and only if

$$\lim_{x \to c^-} f(x) = \lim_{x \to c^+} f(x)$$

So far - Limits give us another way to think about "holes" and vertical asymptotes

- Limits also agree with the function whenever $f(c)$ is defined.

Today we will see that looking at a graph for a function is a quick way to find a limit.

Note if $\lim_{x \to c} f(x) = \pm\infty$, $f(c)$ is undefined

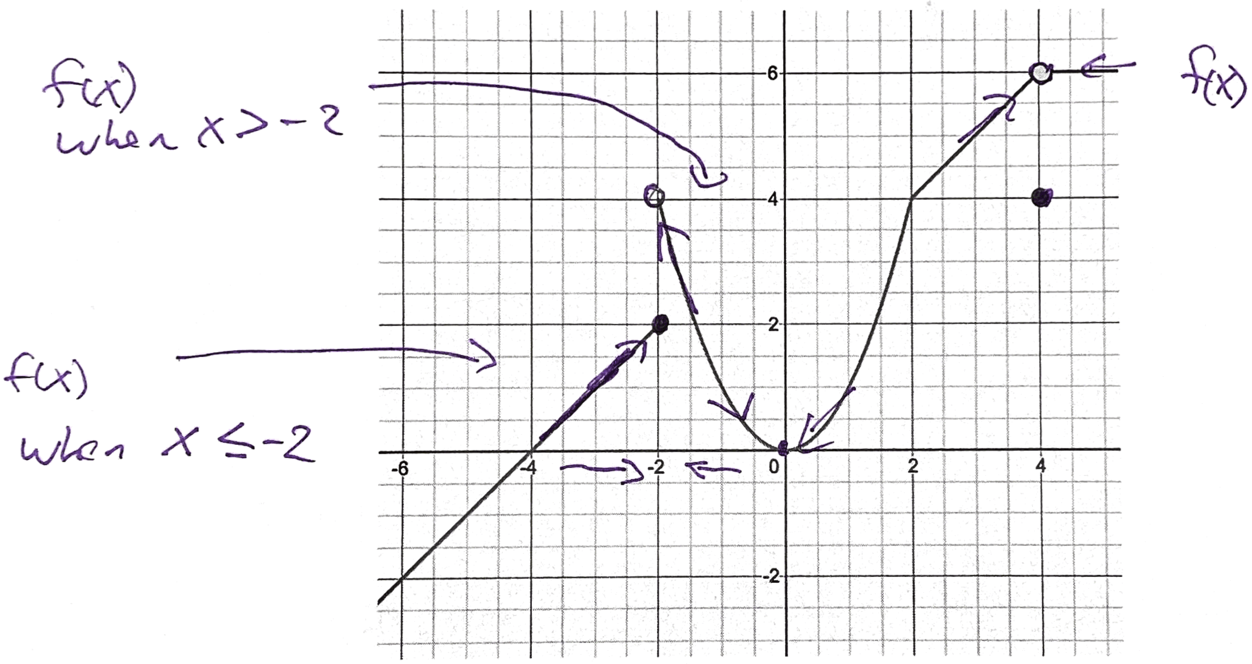
if c is not in the domain of $f(x)$, then its possible for $\lim_{x \to c} f(x) = L$ ← some real number and $f(c)$ to still be undefined.

MA 16010

Lesson 2

14 January 2022

Example 1. Find the following limits from the graph of $f(x)$.



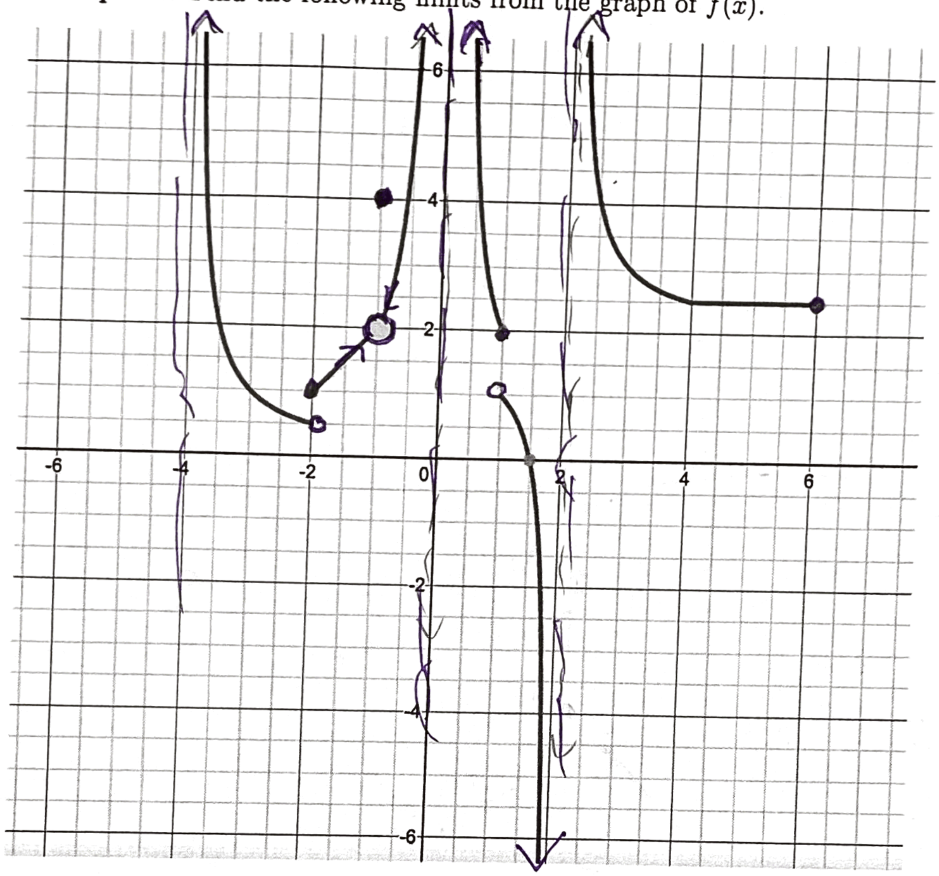
(a) $\lim\limits_{x\to -2^-} f(x) =$ 2 $\qquad \lim\limits_{x\to -2^+} f(x) =$ 4 $\qquad \lim\limits_{x\to -2} f(x) =$ DNE $\qquad f(-2) =$ 2

(b) $\lim\limits_{x\to 0^-} f(x) =$ 0 $\qquad \lim\limits_{x\to 0^+} f(x) =$ 0 $\qquad \lim\limits_{x\to 0} f(x) =$ 0 $\qquad f(0) =$ 0

(c) $\lim\limits_{x\to 4^-} f(x) =$ 6 $\qquad \lim\limits_{x\to 4^+} f(x) =$ 6 $\qquad \lim\limits_{x\to 4} f(x) =$ 6 $\qquad f(4) =$ 4

$\frac{1}{x^2}$ $\frac{1}{x-2}$

Example 2. Find the following limits from the graph of $f(x)$.



All caps

In Lon-cappa

−2	(a) $\lim_{x\to -2^-} f(x) = \frac{1}{2}$	$\lim_{x\to -2^+} f(x) = 1$	$\lim_{x\to -2} f(x) =$ DNE	$f(-2) = 1$
−1	(b) $\lim_{x\to -1^-} f(x) = 2$	$\lim_{x\to -1^+} f(x) = 2$	$\lim_{x\to -1} f(x) = 2$	$f(-1) = 4$
0	(c) $\lim_{x\to 0^-} f(x) = \infty$	$\lim_{x\to 0^+} f(x) = \infty$	$\lim_{x\to 0} f(x) = \infty$	$f(0) =$ Undefined
2	(d) $\lim_{x\to 2^-} f(x) = -\infty$	$\lim_{x\to 2^+} f(x) = \infty$	$\lim_{x\to 2} f(x) =$ DNE	$f(2) =$ undefined

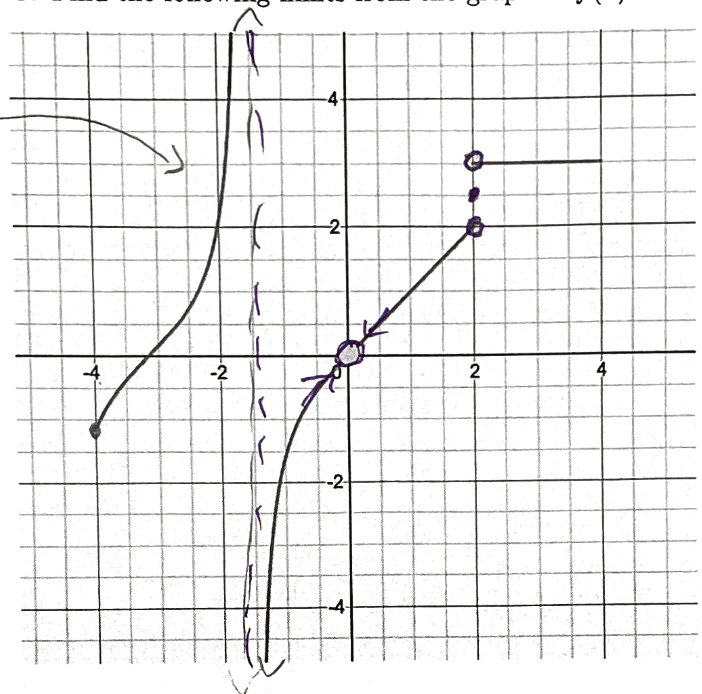
Example 3. Find the following limits from the graph of $f(x)$.

tan(x)

$-4 \le x < 0$

$$\tan x = \frac{\sin x}{\cos x}$$

$$\tan\left(-\frac{\pi}{2}\right) = \frac{\sin\left(\frac{\pi}{2}\right)}{\cos\left(-\frac{\pi}{2}\right)} = \frac{-1}{0}$$



(a) $\lim_{x\to-\frac{\pi}{2}^-} f(x) = \infty$ $\quad\lim_{x\to-\frac{\pi}{2}^+} f(x) = -\infty$ $\quad\lim_{x\to-\frac{\pi}{2}} f(x) =$ DNE $\quad f(-\frac{\pi}{2}) =$ undefined

(b) $\lim_{x\to0^-} f(x) = 0$ $\quad\lim_{x\to0^+} f(x) = 0$ $\quad\lim_{x\to0} f(x) = 0$ $\quad f(0) =$ undefined

(c) $\lim_{x\to2^-} f(x) = 2$ $\quad\lim_{x\to2^+} f(x) = 3$ $\quad\lim_{x\to2} f(x) =$ DNE $\quad f(2) = 2.5$

We will use limits to build the fundamental tools in Calculus.

- Derivatives
- Integrals

Another type of limit, an infinite limit

$$\lim_{x \to \infty} f(x)$$

Lets us understand a lot of natural process.

- Continuous compound interest

$$\lim_{n \to \infty} \left(1 + \frac{1}{n}\right)^n = e^n$$

$A(t) = p.e^{rt}$ is a limit of $A(t) = p\left(1 + \frac{r}{n}\right)^{rn}$

Continuous compound interest

Discrete compound interest