04.11.22

# Lesson 30

Last time we saw the Left and Right Riemann sums were good approximations of the area under a curve on $[a,b]$

$$L_n = \sum_{i=0}^{n-1} f(x_i)\Delta x \longrightarrow \int_a^b f\,dx$$

$$R_n = \sum_{i=1}^{n} f(x_i)\,\Delta x \longrightarrow \int_a^b f\,dx$$

$\Delta x = \dfrac{b-a}{n}$ ← number of rectangles

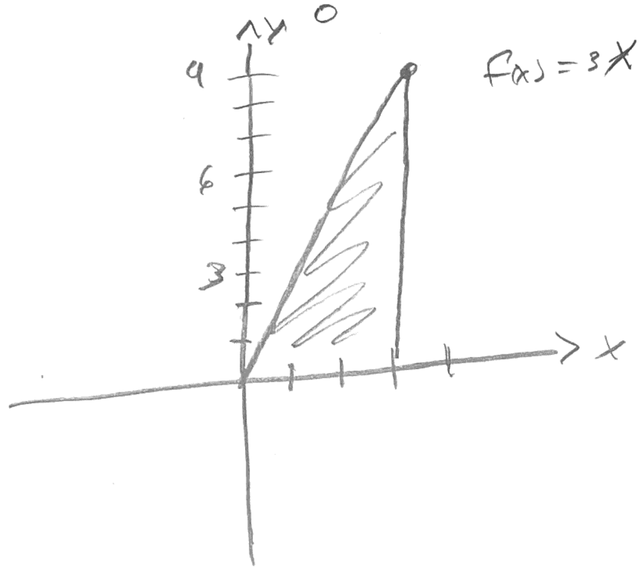
So as we increase the number of rectangles $\Delta x \to dx$ and the Riemann sums approach the definite integral over the interval $[a,b]$.

$$\int_a^b f(x)\,dx$$

<u>definite integral</u> which represents the area under the curve over the interval $[a,b]$.

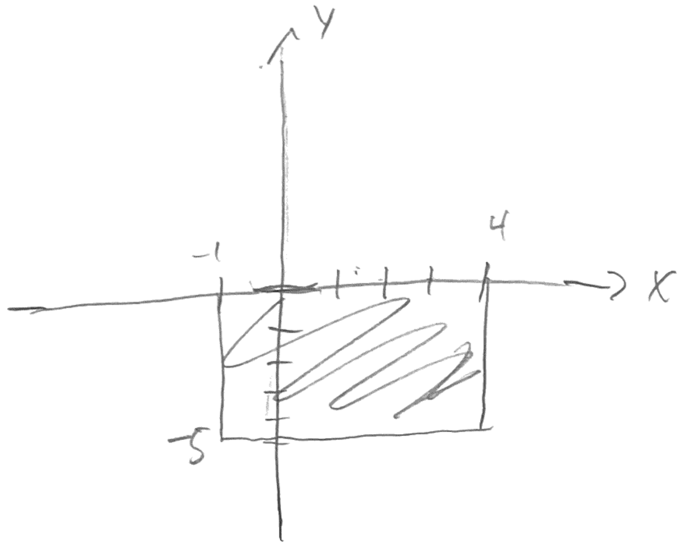
If the area under a curve looks like a familiar shape, we can just calculate the area with a geometric formula.

<u>Example</u>: Evaluate $\int_0^3 3x\,dx$ using geometric formulas.




$$\int_0^3 3x\,dx = \underbrace{\tfrac{1}{2}bh}_{\text{Area of triangle}} = \tfrac{1}{2}\,3(9) = \tfrac{27}{2}$$

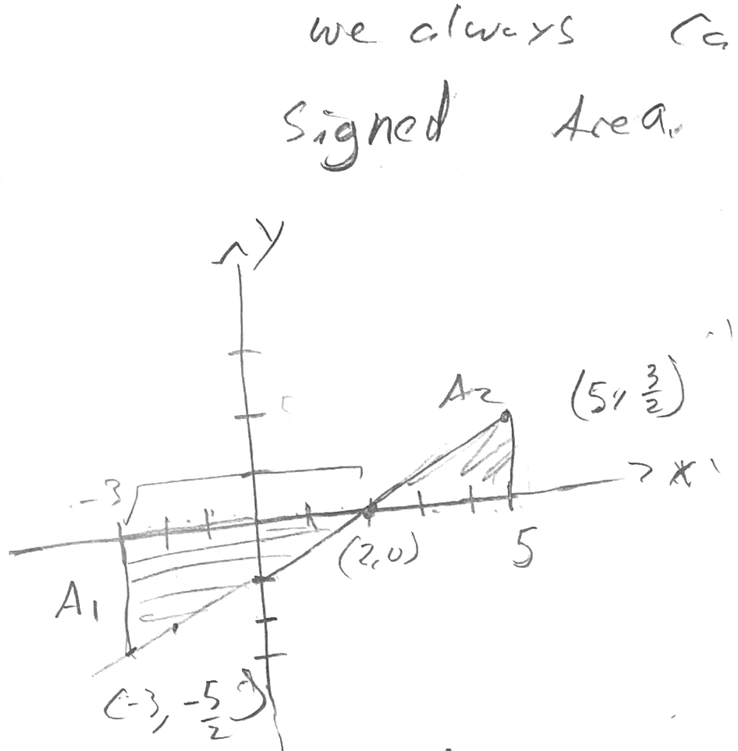
Example 2 Evaluate (a) $\int_{-1}^{4} -5\,dx$ using geometric formulas.




$$\int_{-1}^{4} -5\,dx = 5(-5) = -25$$

negative sign because we always calculate signed Area.

(b) $\int_{-3}^{5} \left(\frac{1}{2}x - 1\right) dx$




$f(x) = \frac{1}{2}x - 1$

$f(5) = \frac{3}{2}$

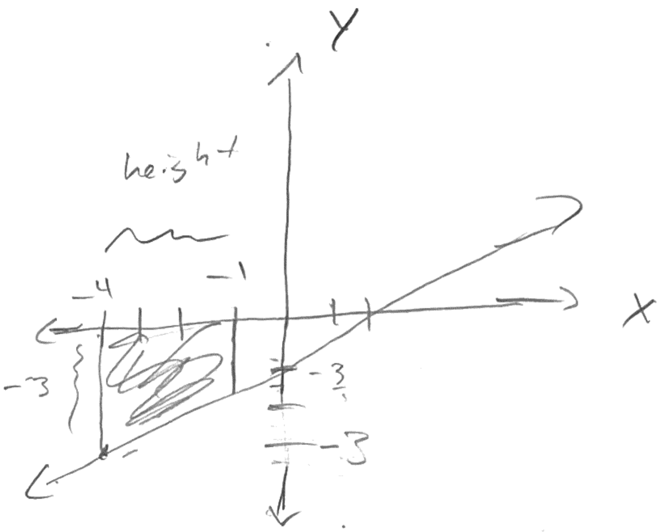
$f(-3) = -\frac{5}{2}$

$$\int_{-3}^{5} \left(\frac{1}{2}x - 1\right) dx = \underbrace{\frac{1}{2}(5)\left(-\frac{5}{2}\right)}_{A_1} + \underbrace{\frac{1}{2}(3)\frac{3}{2}}_{A_2}$$

$$= -\frac{25}{4} + \frac{9}{4} = -\frac{16}{4} = -4$$

(c) Evaluate using geometric formulas

$$\int_{-4}^{-1} \frac{1}{2}x - 1$$




Trapezoid (sideways)

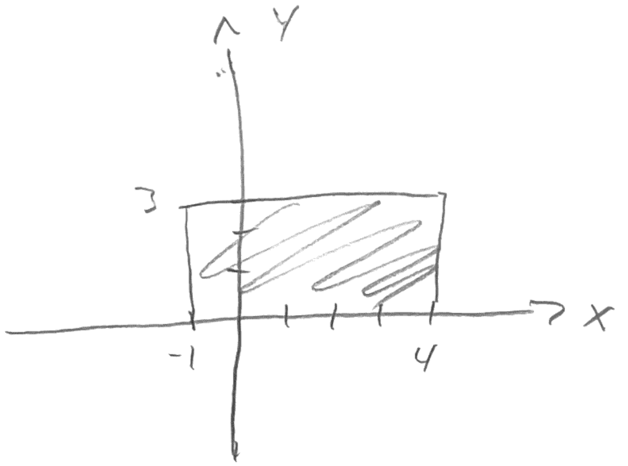
$$= \frac{1}{2}(\text{base } 1 + \text{base } 2) \text{ height}$$

$$= \frac{1}{2}\left(-\frac{3}{2} + -3\right) 3 = -\frac{9}{4}(3) = -\frac{27}{4}$$

or $\frac{1}{2}\Big(\underbrace{\left(\tfrac{1}{2}(-1)-1\right)}_{\text{base } 1} + \underbrace{\tfrac{1}{2}(-4)-1}_{\text{base } 2}\Big) \cdot \underbrace{(4-1)}_{\text{height}}$

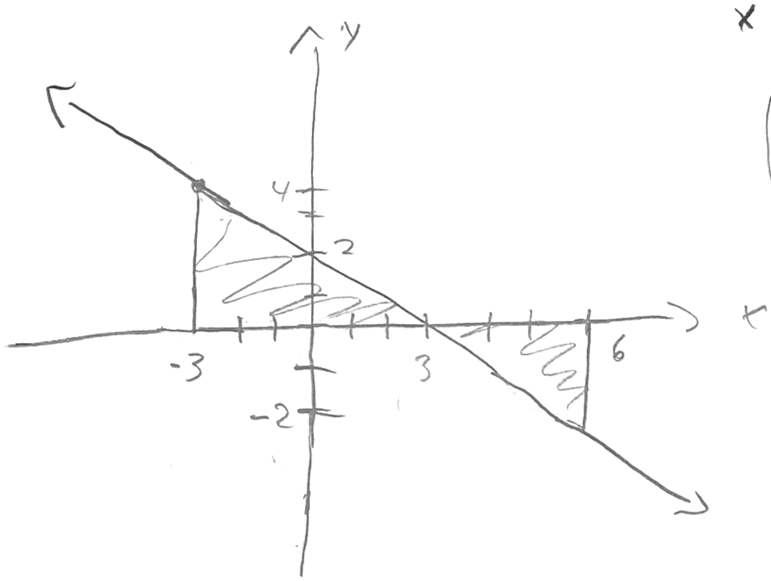
Example 3 write the integral represented by the graph

(a)




$$\int_{-1}^{4} 3\,dx$$

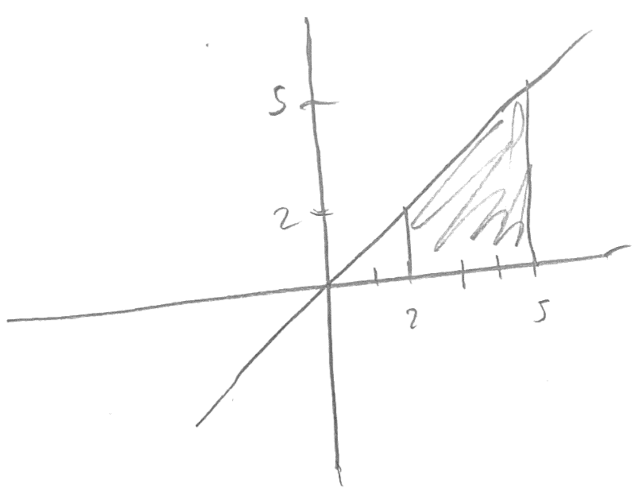
(b)




x values

$$\int_{3}^{6} -\frac{2}{3}x + 2$$

(c)




$$\int_{2}^{5} x\,dx$$

Properties of Definite Integrals:

$$\int_a^a f(x)\,dx = 0$$

$$\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$$

$$\int_a^b k\,f(x)\,dx = k\int_a^b f(x)\,dx$$

$$\int_a^b [f(x) + g(x)]\,dx = \int_a^b f(x)\,dx + \int_a^b g(x)\,dx$$

$$\int_a^b [f(x) - g(x)]\,dx = \int_a^b f(x)\,dx - \int_a^b g(x)\,dx$$

$$\int_a^c f(x)\,dx = \int_a^b f(x)\,dx + \int_b^c f(x)\,dx$$