04.13.22

Lesson 31

Properties of Definite integrals

- Heres our properties again, where a, b, c and k are constants

(i) $\int_a^a f(x)\,dx = 0$ (ii) $\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$

(iii) $\int_a^b k f(x)\,dx = k \int_a^b f(x)\,dx$

(iv) $\int_a^b [f(x) \pm g(x)]\,dx = \int_a^b f(x)\,dx \pm \int_a^b g(x)\,dx$

(v) $\int_a^c f(x)\,dx = \int_a^b f(x)\,dx + \int_b^c f(x)\,dx$

Example 1 Given $\int_3^7 4x^2\,dx = \frac{1264}{3}$ Find:

(a) $\int_7^3 4x^2\,dx = -1264$ by property (ii)

(b) $\int_3^7 12x^2\,dx = 1264$ by (iii)

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Example 2 If $\int_{-2}^{4} f(x)\,dx = 6$, $\int_{-2}^{4} g(x)\,dx = 8$

and $\int_{-2}^{4} h(x)\,dx = 3$, Find

$$\int_{-2}^{4} (5 f(x) + 3 g(x) + 4 h(x)\,dx).$$

Solution

$$\int_{-2}^{4} (5 f(x) + 3 g(x) + 4 h(x))\,dx$$

$$= 5\int_{-2}^{4} f(x)\,dx + 3\int_{-2}^{4} g(x) + 4\int_{-2}^{4} h(x)\,dx$$

$$= 5(6) + 3(8) + 4(3)$$

$$= 30 + 24 + 12 = 66$$

Example 3 If $\int_{-6}^{4} f(t)\,dt = 15$ and $\int_{0}^{4} f(t)\,dt = 20$, find $\int_{-6}^{0} f(t)\,dt$.

Solution

$$\int_{-6}^{4} f(t)\,dt - \int_{0}^{4} f(t)\,dt = \int_{-6}^{0} f(t)\,dt$$

So, $\int_{-6}^{0} f(t)\,dt = 15 - 20 = -5$

Example 4 Given $\int_{a}^{6} g(x)\,dx = 3$ and $\int_{a}^{c} g(x)\,dx = 7\int_{a}^{6} g(x)\,dx$

Find $\int_{b}^{c} g(x)\,dx$.

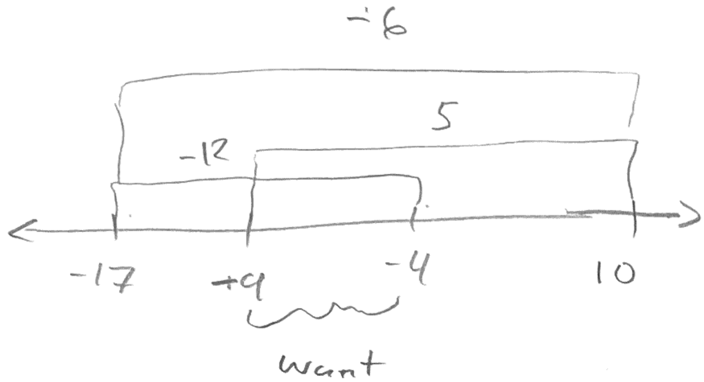
Solution

$$\begin{aligned}\int_{6}^{c} g(x)\,dx &= \int_{a}^{c} g(x)\,dx - \int_{a}^{6} g(x)\,dx \\ &= 7\int_{a}^{6} g(x)\,dx - \int_{a}^{6} g(x)\,dx \\ &= 6\int_{a}^{6} g(x)\,dx\end{aligned}$$

<u>Example 5</u> Given $\int_{-9}^{10} f(x)\,dx = 5$, $\int_{-4}^{-17} f(x)\,dx = 12$

and $\int_{-17}^{10} f(x)\,dx = -6$. Find $\int_{-9}^{-4} f(x)\,dx$

<u>Solution</u>



$$\int_{-17}^{10} f(x)\,dx + \int_{-9}^{-4} f(x)\,dx = \int_{-17}^{-4} f(x)\,dx + \int_{-9}^{10} f(x)\,dx$$

(B/c $\int_{-9}^{4}$ gets counted twice on the RHS!)

Becomes

$$-6 + \int_{-9}^{-4} f(x)\,dx = -12 + 5$$

So, $$\int_{-9}^{-4} f(x)\,dx = -12 + 5 + 6 = -1$$