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Lesson 33

Fundamental Theorem of Calculus

Today we have applications of the Fundamental Theorem of Calculus.

<u>Example 1</u> The velocity function, in meters per minute, of a particle moving along a straight line is

$$v(t) = 7t - 2$$

where t is time in minutes.

(a) Find the displacement of the particle from 3 to 7 minutes

<u>Solution</u>

Displacement comes from position which is the integral of velocity

$$\int_3^7 v(t)\, dt = \underbrace{s(7) - s(3)}_{\text{displacement between 3 and 7 minutes.}}$$

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Example 1 ... Continued

$$\int_3^7 (7t-2)\,dt = \left(\frac{7t^2}{2} - 2t\right)\Big|_3^7$$

$$= \left(\frac{7^3}{2} - 14\right) - \left(\frac{7(3)^2}{2} - 6\right)$$

$$= \frac{343}{2} - 14 - \frac{63}{2} + 6$$

$$= 132$$

(1b) Find the time t when the displacement is zero after the particle starts moving?

$$\int_0^x v(t)\,dt = 0$$

$$\int_0^x (7t-2)\,dt = \left(\frac{7}{2}t^2 - 2t\right)\Big|_0^x = 0$$

$$= \frac{7}{2}x^2 - 2x = 0$$

$$\frac{7}{2}x^2 = 2x$$

$$x = \frac{4}{7}$$

The displacement is zero $4/7$ minutes after the particle starts moving.

Example 2 The growth rate of the population of a country is

$$p'(t) = \sqrt{t}\,(5000t + 1000)$$

where t is time in years.

Q: How much does the population increase from $t=2$ year to $t=4$.

Solution:

$$P(t) = \int_2^4 \sqrt{t}\,(5000t + 1000)\,dt$$

$$= \int_2^4 (5000\,t^{3/2} + 1000\,t^{1/2})\,dt$$

$$= \left(5000\,t^{5/2}\left(\frac{2}{5}\right) + 1000\,t^{3/2}\left(\frac{2}{3}\right)\right)\Big|_2^4$$

$$= \left(2000\,(4)^{5/2} + \frac{2000}{3}\,(4)^{3/2}\right) - \left(2000\,(2)^{5/2} + \frac{2000}{3}\,(2)^{3/2}\right)$$

$$= 2000(32) + \frac{2000}{3}(8) - 2000\,(2)^{5/2} - \frac{2000}{3}\,(2)^{3/2}$$

$$= 56134$$

Example 3 The acceleration of a train t seconds after the driver steps on the break, before the car comes to a full stop, is

$$a(t) = -(t-9)^2 \text{ mph per second.}$$

Whats the decrease in velocity in mph 5 seconds after the brake is applied?

Solution

$$\int_0^5 a(t)\,dt = \underbrace{V(5) - V(0)}_{\text{decrease in velocity}}$$

$$\int_0^5 -(t-9)^2\,dt = \int_0^5 (-t^2 + 18t - 81)\,dt$$

$$= -\frac{t^3}{3} + 9t^2 - 81t \Big|_0^5$$

$$= -\frac{(5)^3}{3} + 9(5)^2 - 81(5)$$

$$= \frac{-125}{3} + 9(25) - 81(5)$$

$$= -\frac{665}{3} \approx -221.667$$

The decrease in velocity is 221.667 mph 5 seconds after the brake is applied.