04.24.22

Lesson 36

Exponential Decay

- If we have an exponential growth model with a negative growth rate it is called <u>exponential decay</u>. Radioactive decay is the most common example

- Exponential decay still comes from a differential equation.

$$\frac{dy}{dt} = -3y \quad \text{and} \quad y(4) = 100.$$

Find $y(t)$.

$$y = ce^{-3t}$$

$$y(4) = ce^{-3(4)} = 100$$

$$c = \frac{100}{e^{-12}} = e^{12}100$$

$$y(t) = 100e^{-3t+12}$$

$$= 100e^{3(4-t)}$$

Example! The radioactive isotope ^{200}Ra (Radium) has a half-life of approximately 1375 years. There are 60 g of ^{220}Ra now.

(a) How many grams remain after 1500 years?

(b) How many grams remain after 15000 years?

Solution

- start with the general equation

$$y = 35e^{Kt}$$

$c = 35$ from initial condition

- use half-life: $y(1375) = 60e^{1375K} = 30$

$$e^{1375K} = \frac{1}{2}$$

$$1375K = \ln\left(\tfrac{1}{2}\right)$$

$$K = \frac{\ln\left(\frac{1}{2}\right)}{1375} \approx -0.000504$$

(a) $$y(1500) = 60e^{\overbrace{\frac{\ln(\frac{1}{2})}{1375}}^{K}(1500)} = 28.168 \text{ grams}$$

← Round at this step, not for K.

(b) $$y(15000) = 60e^{\frac{\ln(\frac{1}{2})(15000)}{1375}} = 0.031 \text{ grams}$$

So after 15000 years the radio active material is still around!

<u>Example 2</u> The radio active istope ^{239}Pu has a half-life of approximately 24100 years. After 1100 years, there are 5.5 g of ^{239}Pu remaining.

(a) What is the initial quantity?

(b) How much remains after 11000 years?

<u>Solution</u>

We know $y(24100) = Ce^{24100k} = \frac{1}{2}C$

because this is the half life. So

$$e^{24100k} = \frac{1}{2}$$

$$24100k = \ln\left(\tfrac{1}{2}\right)$$

$$k = \frac{\ln\left(\frac{1}{2}\right)}{24100}$$

So far, $y(t) = Ce^{\frac{\ln\left(\frac{1}{2}\right)}{24100}t}$

Example 2 ... continued

Now we can use the initial condition to find C. $Y(1100) = 5.5$

So

$$Y(1100) = Ce^{1100\left(\frac{\ln\frac{1}{2}}{24100}\right)} = 5.5$$

$$C = 5.5\, e^{-1100\frac{\ln\left(\frac{1}{2}\right)}{24100}}$$

$$= 5.677$$ (Round to 3 places)

(a) So the initial quantity is 5.677 grams

(b) $$Y(11000) = 5.677e^{11000\frac{\ln\frac{1}{2}}{24100}}$$

(Round to 3 places)

$$= 4.137$$

Example 3 Radioactive radium has a half-life of approximately 1599 years. What percent of a given amount remains after 900 years?

Solution $$Y(1599) = Ce^{1599k} = \frac{1}{2}C$$

$$e^{1599k} = \frac{1}{2}$$

$$k = \ln\left(\tfrac{1}{2}\right)\frac{1}{1599}$$

Example 3 ...continued

So $$y = C e^{\frac{\ln\left(\frac{1}{2}\right)}{1599} t}$$

To answer our question, plug in $t = 900$.

$$y(800) = C e^{\frac{\ln\left(\frac{1}{2}\right)}{1599}(900)}$$

$$= C(0.677)$$

So $\approx 70.7\%$ of a given amount remains after 900 years.

Note the actual amount is unknown, just how much is left compared to what we started with.