

01.21.22

Lesson 5 Continuity

- A function is Continuous at a point $x=c$ if you can draw the graph without picking up your pencil at c .

- More precisely, the following conditions must be met.

1. $f(c)$ is defined
2. $\lim_{x \rightarrow c} f(x)$ exists
3. $\lim_{x \rightarrow c} f(x) = f(c)$

← important! It's not enough for the limit to exist

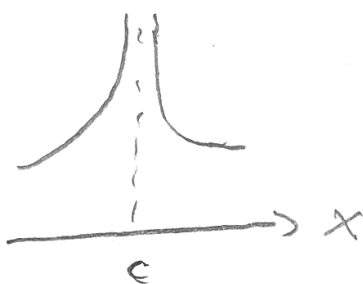
- Types of Discontinuities



hole

$f(c)$ undefined

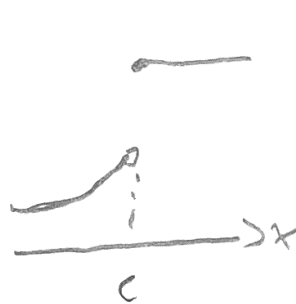
(breaks 1 & 3)



vertical asymptote at $x=c$

$f(c)$ undefined

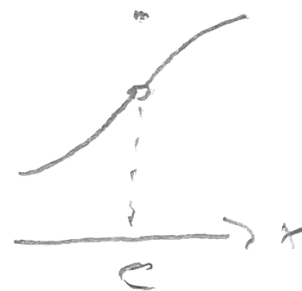
(breaks 1 & 3)



jump

$\lim_{x \rightarrow c} f(x)$ DNE

(breaks 2)



hole

$\lim_{x \rightarrow c} f(x) \neq f(c)$

(breaks 3)

Example 1 Let $f(x) = \frac{x^2 - 3x - 10}{x^2 + 9x + 14} = \frac{(x-5)(x+2)}{(x+7)(x+2)}$

Where is $f(x)$ discontinuous?

What type of discontinuities are these?

discontinuous at $x = -2$, hole (breaks 1 & 3)

$x = -7$, vertical asymptote (breaks 1 & 2)

Idea: Describing points as discontinuous lets us categorize all the "problem points" I have mentioned

Example 2 Let $f(x) = \begin{cases} 2x+3 & x \leq -2 \\ x^2+2 & -2 < x \leq 0 \\ 2 & 0 < x < 2 \\ 3 & x = 2 \\ x^2-2 & x > 2 \end{cases}$

Find and categorize the discontinuities of $f(x)$

discontinuities at

$x = -2$ b/c $\lim_{x \rightarrow -2^-} f(x) = 1 \neq \lim_{x \rightarrow -2^+} f(x) = 6$

(breaks 2)

jump discontinuity

continuous

Example 2 ... continued

$$- x=2 \quad \lim_{x \rightarrow 2} f(x) = 2 \neq f(2) = 3$$

(breaks 3) hole

- Note: $f(x)$ is continuous at $x=0$

$$\lim_{x \rightarrow 0} f(x) = 2 = f(0)$$

Example 3 Let $f(x) = \begin{cases} 3e^{-4x} & x \leq 0 \\ 3 \cos(x) & 0 < x \leq \pi \\ \frac{4}{\pi}x & x > \pi \end{cases}$

Find where $f(x)$ is discontinuous and say what type of discontinuity it is

discontinuous at

$$x = \pi \quad \text{since, } \lim_{x \rightarrow \pi^-} f(x) = -3 \neq \lim_{x \rightarrow \pi^+} f(x) = 4$$

since $f(\pi) = -3$, only 2 is broken

Note: $f(x)$ is continuous at $x=0$ since

$$\lim_{x \rightarrow 0^-} f(x) = \lim_{x \rightarrow 0^+} f(x) = 3 = f(0).$$