Review

e, ln, negative exponents

- $f(x) = e^x$ and $g(x) = \ln(x)$ are inverses of each other so $e^{\ln x} = x$ and $\ln e^x = x$

- exponent rules:

(1) $e^a \cdot e^b = e^{a+b}$

(2) $(e^a)^b = e^{a \cdot b}$

(3) $e^{-a} = \frac{1}{e^a}$

a & b can be variables (x) or numbers (7, $\frac{1}{2}$)

- logarithm rules:

(1) $\ln(ab) = \ln(a) + \ln(b)$

(2) $\ln\left(\frac{a}{b}\right) = \ln(a) - \ln(b)$

(3) $\ln(a^b) = b \ln(a)$ $\Leftarrow$ Power rule for log's used on quiz

limits

$\lim_{x \to c^-} f(x)$ pick $x < c$ to test

$\lim_{x \to c^+} f(x)$ pick $x > c$ to test

Always exist in our class

$\lim_{x \to c} f(x)$ exists iff

$$\lim_{x \to c^-} f(x) = \lim_{x \to c^+} f(x)$$

step 1 plug in $x = c$

if $\frac{\text{number}}{0}$ step 2 test i.e. $-2 < -1.99$

Piecewise $x \to c^-$ means plug into $x \leq c$

$x \to c^+$ means plug into $x \geq c$

1.26.22

Lesson 7

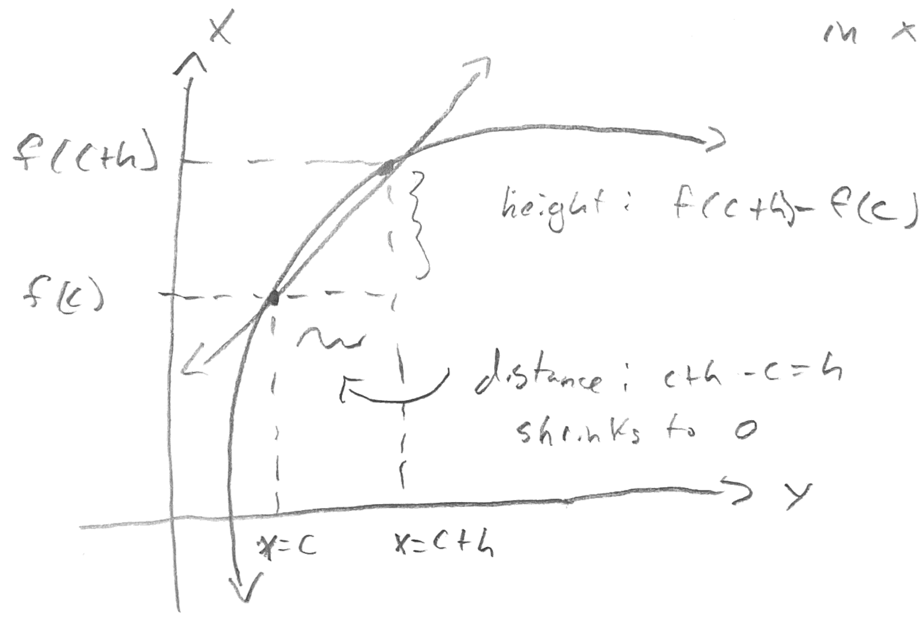
Baisc rules of Differentiation

Last time: When $y = f(x)$, the derivative of $f(x)$ is also a function $f'(x)$.

- $f'(x)$ is the slope of the tangent line to $f(x)$. It changes from point to point, so when $x=c$ $f'(c)$ is the slope of the tangent line at $x=c$.
- $f'(x)$ is a limit, we shrink the distance between the two points in a slope formula by letting $h \to 0$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

Δx is called "change in x", read "delta x"



Rules of Differentiation

Notation: $\frac{d}{dx}[f(x)] = f'(x)$

1) $\frac{d}{dx}[c] = 0$; For any constant like 7 or π the derivative is always 0.

2) $\frac{d}{dx}[c\, f(x)] = c \frac{d}{dx}[f(x)]$; When a function is multiplied by a constant taking the derivative does not effect it

3) $\frac{d}{dx}[f(x) + g(x)] = \frac{d}{dx}[f(x)] + \frac{d}{dx}[g(x)]$

4) $\frac{d}{dx}[f(x) - g(x)] = \frac{d}{dx}[f(x)] - \frac{d}{dx}[g(x)]$

The Power Rule

$$\frac{d}{dx}[x^n] = n\,x^{n-1}$$

n is any number

Example 1 Find $f'(x)$ of the following

(a) $f(x) = x^5 + x^{-4}$

$$f'(x) = 5x^{5-1} + (-4)x^{-4-1}$$
$$= 5x^4 - 4x^{-5}$$

(b) $f(x) = 2x^{\frac{1}{2}}$

$$f'(x) = 2\left(\tfrac{1}{2}x^{\frac{1}{2}-1}\right)$$
$$= x^{-\frac{1}{2}}$$
$$= \frac{1}{\sqrt{x}}$$

Example 2 Find the derivatives

(a) $f(x) = 2x^{\frac{\pi}{2}} - 3x^{0.4}$

$f'(x) = 2\left(\frac{\pi}{2} x^{\left(\frac{\pi}{2}-1\right)}\right) - 3\left(0.4x^{0.4-1}\right)$

$= \pi x^{\frac{\pi}{2}-1} - 1.2\, x^{-0.6}$

$= \pi x^{\frac{\pi}{2}-1} - \frac{1.2}{x^{0.6}}$

(b) $y = \frac{11}{\sqrt[4]{x}} + \sqrt[3]{x^2}$

$y = 11x^{-1/4} + x^{2/3}$

$y' = -\frac{11}{4} x^{-1/4-1} + \frac{2}{3} x^{\frac{2}{3}-1}$

$= -\frac{11}{4} x^{-5/4} + \frac{2}{3} x^{-1/3}$

$= \frac{-11}{4x^{5/4}} + \frac{2}{3x^{1/3}}$

Example 3 Find $\left.\frac{dy}{dx}\right|_{x=2}$

$y = x^2(-3x^3 + 5x + 4x^{-2})$

$y = -3x^5 + 5x^3 + 4x^{2-2}$

$= -3x^5 + 5x^3 + 4$ $(x^0 = 1)$

$y' = -15x^4 + 15x^2$ remember 4 is a constant and derives to 0.

$\left.\frac{dy}{dx}\right|_{x=2} = -15(2)^4 + 15(2)^2 = -180$

Example 4 Find $f'(1)$

$$f(x) = \frac{x^{2.7} - 3x^3}{\sqrt{x}}$$

$$f(x) = \frac{x^{2.7}}{x^{1/2}} - \frac{3x^3}{x^{1/2}} = x^{2.7-\frac{1}{2}} - 3x^{3-1/2}$$

$$= x^{2.2} - 3x^{5/2}$$

$$f'(x) = 2.2\,x^{(2.2-1)} - 3\left(\tfrac{5}{2}x^{(5/2-1)}\right)$$

$$= 2.2\,x^{1.2} - \frac{15}{2}x^{3/2}$$

$$f'(1) = 2.2 - \frac{15}{2} = \frac{22}{10} - \frac{75}{10} = -\frac{53}{10} = \boxed{-5.3}$$

Example 5 Find the equation of the tangent line to the graph of $f(x) = \frac{x^3}{5} - \frac{4}{x}$ at $x = -1$

$$f(x) = \frac{1}{5}x^3 - 4x^{-1}$$

$$f'(x) = \frac{3}{5}x^2 + 4x^{-2}$$

$$f'(-1) = \frac{3}{5}(-1)^2 + \frac{4}{(-1)^2}$$

$$= \frac{3}{5} + 4 = \frac{23}{5}$$

So $m = \frac{23}{5}$

eq of line:

$$(y - f(-1)) = \underbrace{\frac{23}{5}}_{m}(x - (-1))$$

$$f(-1) = \frac{-1}{5} + 4 = \frac{19}{5}$$

$$\left(y - \frac{19}{5}\right) = \frac{23}{5}x + \frac{23}{5}$$

$$\boxed{y = \frac{23}{5}x + \frac{42}{5}}$$

Some Special derivatives

$$\frac{d}{dx}[\sin x] = \cos x$$

$$\frac{d}{dx}[\cos x] = -\sin x$$

$$\frac{d}{dx}[e^x] = e^x$$ ← Note: e^x is the only function which is its own derivative!

Example 6 Find the following derivatives and evaluate $f(x)$ at $x = \pi$

$x = 4$

(a) $f(x) = \pi\cos(x) - 3\sin(x)$

$f'(x) = \pi(-\sin(x)) - 3(\cos(x))$

$f'(\pi) = -\pi\sin(\pi) - 3\cos(\pi)$

$= 0 - 3$

$= -3$

(b) $f(x) = 2e^x - \sqrt{x}$

$f'(x) = 2e^x - \frac{1}{2}x^{-\frac{1}{2}}$

$f'(4) = 2e^4 - \frac{1}{2\sqrt{4}}$

$= 2e^4 - \frac{1}{4}$

Explinations

$$\frac{d}{dx}[\sin x] = \lim_{h\to 0} \frac{\sin(x+h) - \sin(x)}{h}$$

By sum/diff. trig identity

$$= \lim_{h\to 0} \frac{\sin(x)\cos(h) + \cos(x)\sin(h) - \sin(x)}{h}$$

$$= \lim_{h\to 0}\left(\frac{\sin(x)\cos(h) - \sin(x)}{h} + \frac{\cos(x)\sin(h)}{h}\right)$$

$$= \sin(x)\underbrace{\lim_{h\to 0}\frac{\cos(h)-1}{h}}_{=0} + \cos(x)\underbrace{\lim_{h\to 0}\frac{\sin(h)}{h}}_{=1}$$

from limit chapter

$$= \sin(x)\cdot 0 + \cos x \cdot 1$$

$$= \cos x$$

$$\frac{d}{dx}[e^x] = \lim_{h\to 0}\frac{e^{x+h} - e^x}{h} = \lim_{h\to 0}\frac{e^x \cdot e^h - e^x}{h}$$

$$= e^x \lim_{h\to 0}\underbrace{\frac{e^h - 1}{h}}_{=1}$$

from limit HW.

$$= e^x$$