01.28.22

Lesson 8

Instantaneous Rate of Change

Last time

Power rule: $\frac{d}{dx}[x^a] = a x^{a-1}$ a, any real number

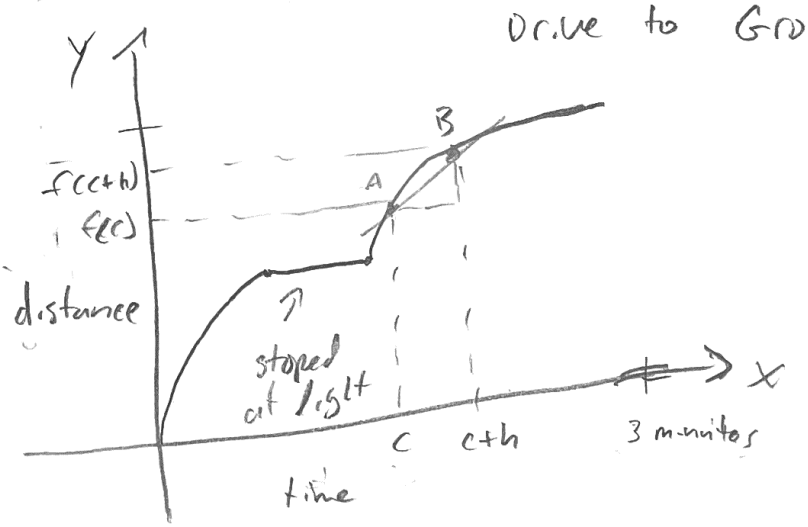
$\frac{d}{dx}[\cos x] = -\sin x$

$\frac{d}{dx}[\sin x] = \cos x$

$\frac{d}{dx}[e^x] = e^x$

Today derivative as rate of change

Drive to Grocery Store



$$\frac{f(x+h) - f(x)}{h}$$

This is called the average rate of change from point A to B

- Letting $h \to 0$ we get the instantaneous rate of change

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

is the instantaneous rate of change

Position distance from starting point $s(t)$ or $h(t)$

Velocity speed of an object with respect to direction. $v(t) = s'(t)$

Example 1: A delivery truck's position in meters is given by $s(t) = 5t^2 + 7t + 11$ for the first two minutes of the drive.

Find the velocity function

$$v(t) = s'(t) = 10t + 7$$

what is the velocity after the first minute?

$$v(1) = 10(1) + 7 = 17$$

Example 2 A wild fire in California is in the shape of a circle and Spreading. What is the rate of change of the area of the wild fire with respect to its radius when $r = 7$ miles.

$$A(r) = \pi r^2$$

$$A'(r) = 2\pi r$$

$$A'(7) = 2\pi(7) = 14\pi \approx 43.98 \text{ miles}$$

Example 3 During a four day period a virus spreads in a community and the number of total cases (in hundreds of people) can be modeled by

$$f(x) = \frac{1}{3}e^x - \frac{1}{5}x^3$$

where x is numbers of days

What is the rate of change of total cases on the third day

$$f'(x) = \frac{1}{3}e^x - \frac{3}{5}x^2$$

$$f'(3) = \frac{1}{3}e^3 - \frac{3}{5}(9) = \frac{e^3}{3} - \frac{27}{5} = 1.29$$

so 129 cases/day

Example 4 The position of a particle moving in a straight line is given (in meters) by

$$S(t) = -11t^3 + 13t + 21$$

where t is measured in seconds

(a) At what time is the velocity zero?

$$V(t) = -33t^2 + 13$$

set equal to zero and solve for time

$$-33t^2 + 13 = 0$$

$$-33t^2 = -13$$

$$t^2 = \frac{13}{33} \qquad t = \pm\sqrt{\frac{13}{33}} \approx 0.627$$

(b) What is the position of the particle at this moment? Round to two decimal places

$$S\left(\sqrt{\frac{13}{33}}\right) \approx 26.43$$

Example 5 The population of polar bears on a certain ice cap since the year 2010 can be modeled as

$$P(t) = -20x^2 + 300x + 7500$$

where $t=0$ corresponds to the year 1990.

In which year is the population increasing at a rate of 80 polar bears per year?

$$P'(t) = -40t + 300 = 80$$

$$-40t = -220$$

$$t = \frac{-220}{-40} = 5.5$$

so in 2015 (because of how we measure time)