Review

$* \quad (x+h)^2 = (x+h)(x+h) \quad *$

$$= x^2 + 2hx + h^2$$

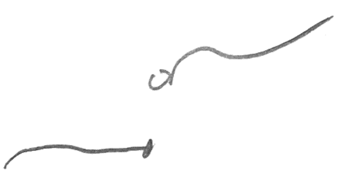
- A function is <u>continuous</u> at $x=c$ is

  (1) $f(c)$ Exists

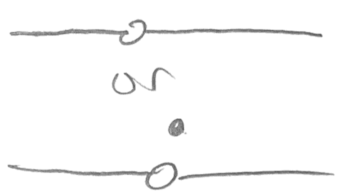
  (2) $\lim_{x \to c} f(x)$ Exists

  (3) $\lim_{x \to c} f(x) = f(c)$

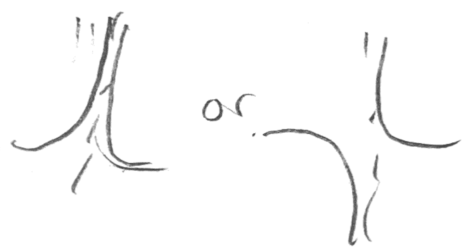
- A function is <u>discontinuous</u> at c if any of the above conditions are broken



Jump



hole



Vertical Asymptote

Lesson 9

Product Rule

Last time

Derivative as instantaneous rate of change
(or rate of change)

$$\frac{dy}{dx} \text{ or } \frac{df}{dx}$$
change in the function output
} ratio "rate of change"
change in x

rate of change of population
Velocity derivative of position

Today Product rule

How to derive $F(x) = f(x)\, g(x)$?

Product rule: $\frac{d}{dx}[f(x)g(x)] = f'(x)\, g(x) + f(x)\, g'(x)$

Example 1 find $f'(x)$ when

$$f(x) = 2x^3(3x^2 - 5)$$

two ways:

old way: $f(x) = 6x^5 - 10x^3$

$$f'(x) = 30x^4 - 30x^2 = 30x^2(x^2 - 1)$$

Example 1 ... cont.

New way

$$f(x) = \underbrace{2x^3}_{h(x)} \underbrace{(3x^2-5)}_{g(x)}$$

$$f'(x) = h'(x)\,g(x) + h(x)\,g'(x)$$

$$= 6x^2(3x^2-5) + 2x^3(6x)$$

$$= 18x^4 - 30x^2 + 12x^4$$

$$= 30x^4 - 30x^2$$

Same answer, but didn't really save time

Example 2 Let $f(x) = (x^7 - 3x)(6x^2+11)$

Find $f'(x)$

Product rule is a little faster now

$$f'(x) = \underbrace{(7x^6-3)}_{h'(x)}\underbrace{(6x^2+11)}_{g(x)} + \underbrace{(x^7-3x)}_{h(x)}\underbrace{(12x)}_{g'(x)}$$

$$= 42x^8 - 18x^2 + 77x^6 - 33 + 12x^8 - 36x^2$$

$$= 54x^8 + 77x^6 - 54x^2 - 33$$

Example 3

Find the derivative of

$$Y = 3e^{x}\cos x - x\sin x \quad \text{and} \quad Y'\left(\tfrac{\pi}{2}\right)$$

Notice. (1) Now we must use the product rule, we don't have any other method

(2) we need to use the product rule twice

$$Y' = \underbrace{3e^{x}}_{f'}\underbrace{\cos x}_{g} + \underbrace{3e^{x}}_{f}\underbrace{(-\sin x)}_{g'} - \left(\underbrace{1}_{f'}\underbrace{\sin x}_{g} + \underbrace{x}_{f}\underbrace{\cos x}_{g'}\right)$$

$$= 3e^{x}\cos x - 3e^{x}\sin x - \sin x + x\cos x$$

$$Y'\left(\tfrac{\pi}{2}\right) = 3e^{\frac{\pi}{2}}\underbrace{\cos\left(\tfrac{\pi}{2}\right)}_{=0} - 3e^{\pi/2}\underbrace{\sin\tfrac{\pi}{2}}_{=1} - \sin\tfrac{\pi}{2} + \tfrac{\pi}{2}\underbrace{\cos\tfrac{\pi}{2}}_{=0}$$

$$= -3e^{\pi/2} - 1$$

Example 4 Let $f(x) = (6\sqrt{x} + 3)\sin x$

Find $f'(x)$

$$f'(x) = \underbrace{\left(6 \tfrac{1}{2} x^{-1/2}\right)}_{h'} \underbrace{\sin x}_{g} + \underbrace{(6\sqrt{x}+3)}_{h} \underbrace{\cos x}_{g'}$$

$$f'(x) = \frac{3\sin x}{\sqrt{x}} + 6\sqrt{x}\cos x + 3\cos x$$

Example 5 Find the x value at which $y = 3x^7 e^x$ has a horizontal tangent line.

$$y' = 3\underbrace{(7x^6)}_{f'} \underbrace{e^x}_{g} + \underbrace{3x^7}_{f} \underbrace{e^x}_{g'}$$

horizontal tangent line has slope $m = 0$

$$y' = 3x^6 e^x (7 + x) = 0$$

set equal to zero

$x = -7$ or $x = 0$

Example 6 Let $y = \cos x\,(e^x + \sin x)$

Find y'.

$$y' = -\sin x\,(e^x + \sin x) + \cos x\,(e^x + \cos x)$$

$$= -e^x \sin x - \sin^2 x + e^x \cos x + \cos^2 x$$