

Quiz 2 Solutions

Problem 1

$$f(x) = \begin{cases} 5x^2 - 4 & x \leq 2 \\ 2x^2 + 1 & 2 < x \leq 3 \end{cases}$$

(a) $\lim_{x \rightarrow 2^-} f(x) = 5(2)^2 - 4 = 16$ (plug into $x \leq 2$ side)

(b) $\lim_{x \rightarrow 2^+} f(x) = 2(2)^2 + 1 = 9$ (plug into $2 < x \leq 3$ side)

(c) $\lim_{x \rightarrow 2} f(x) \text{ DNE}$ (B/c $\lim_{x \rightarrow 2^-} f(x) \neq \lim_{x \rightarrow 2^+} f(x)$)

Problem 2

- $f(x)$ is only discontinuous at $x=2$

B/c $\lim_{x \rightarrow 2} f(x) \text{ DNE}$

- It is a jump discontinuity

B/c the function jumps from 16 to 9 at $x=2$

Problem 3

$$f(x) = x^2 - 3$$

$$\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \rightarrow 0} \frac{\overbrace{(x+h)^2 - 3}^{f(x+h)} - \overbrace{(x^2 - 3)}^{f(x)}}{h}$$

$$= \lim_{h \rightarrow 0} \frac{x^2 + 2hx + h^2 - 3 - x^2 + 3}{h}$$

$$= \lim_{h \rightarrow 0} \frac{2hx + h^2}{h} = \lim_{h \rightarrow 0} \frac{2x + h}{1} = \boxed{2x}$$

let $h \rightarrow 0$
at this step