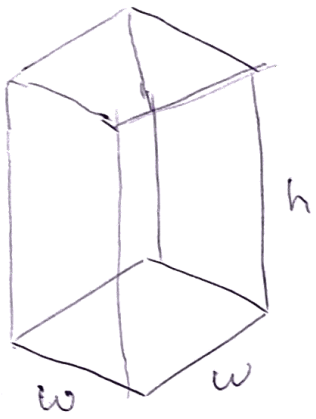

S.A = 2(□ w) + 4(□ h)
w w
top+bottom 4 sides

MA 16010 Quiz 9

Lesson 24-25

30 March 2022

Problem 1. A closed box with a square base has a volume of $1000\,cm^3$. Find the dimensions of the box that requires the least amount of material. How much material is required at the minimum? Make sure to show all of your work.

(a) What is the objective function?

(b) Width = 10
Length = 10
Height = 10

(Needed to get up correct objective function and derive it to get full credit)

Constraint Volume $1000 = w^2 h$

Objective function Surface Area $A = \underbrace{2w^2}_{\text{top+bottom}} + \underbrace{4wh}_{\text{4 sides}}$

- From constraint: $h = \frac{1000}{w^2}$

- Plug in h to A: $A = 2w^2 + 4w\left(\frac{1000}{w^2}\right)$
$= 2w^2 + \frac{4000}{w}$

- optimize A: $A' = 4w - \frac{4000}{w^2} = 0$ ← set

$4w = \frac{4000}{w^2}$

$w^3 = \frac{4000}{4} = 1000$

$w = (1000)^{1/3} = 10$

Domain: $(0, \infty)$ B/c of constraint

By 2nd Der. Test

$A'' = 4 + \frac{8000}{w^3}$,

$A''(10) = 4 + \frac{8000}{10^3} > 0$

So $w=10$ is rel. min and abs. min