LAB 2: A POLLUTED POND

Prior to lab. Read pages 3 and 4 of the enclosed material.

During the lab. The lab assistant will demonstrate the "dfield" command in MATLAB, showing the the following:

(1) How to change the differential equation and the range on S and t. (Use the equation on p. 3 as an example.)
(2) **Key board** input.
(3) The **Replot later** feature in the **Options** menu.
(4) The *Zoom* feature.

This lab refers to the polluted pond described on the last two pages. We will assume that all of the data is exactly as stated on p. 3 *except* no one is dumping trash into the pond. This has the consequences that

(1) The volume of the pond is not changing.
(2) The rate at which water flows out via stream C remains at 1250 cubic meters per day.

Your assignment is to answer the following questions:

(1) Let notation be as on p. 3. Derive the formula

$$\frac{dS}{dt} = 2.5 - .125\,S$$

where $S(t)$ is the amount of salt in the pond at time t.
(2) Use "dfield" to plot the concentration of the pollutant in the pond, assuming that the pond was originally unpolluted. Looking at your graph, what appears to be the limiting concentration of the pollutant in the pond? Why is this expected on physical grounds? Find (mathematically) all equilibria for this equation. How does this relate to your answer to the previous question? (Warning: $S(t)$ reperesents amount of salt, not concentration.)
(3) Find an analytical solution to the above differential equation. Use your answer to prove that the final concentration of the pond will be the same, regardless of the initial concentration.
(4) Let us agree that the pond is totally polluted when the concentration of salt in the pond reaches 99% of its final value. Use your graph to estimate when this occures. Give an answer that is correct to the nearest tenth of a day. Get the graph printed and indicate how you reach your conclusion. (Note: You might need to use the "zoom" feature of dfield.) Now use the analytical solution to answer the same question. Which seems easier?
(5) Suppose that after 10 days, the source of the pollution in stream A is cut off so that only fresh water enters the pond. Use dfield to produce a graph showing the pollution in the following days. Also, find a formula for the

pollution in the following days. Use your formula to determine how long will it take before there is less than .1 kg/1000 cum. of salt disolved in the pond.

Note: the equation in problem 1 will need to be modified to take account of the new conditions. You will also need to determine the pollution level of the pond at the time that stream A is cleaned up.

(6) Suppose that the pond starts out polluted. Specifically, suppose that the initial pollution level is 1 kilogram per 1000 cubic meters. How does this change your answer to question 4? You may use either analytical or graphical analysis to answer this question. However, give an answer that is correct to the nearest tenth of a day.