

## Lab 5

**Prior to lab.** Read the material at the end of this Lab.

**During the lab.** Your assignment is based on the model for the swaying skyscraper described at the end of this lab. You should analyze the data below. Note: The columns  $y_1(t)$  and  $y_2(t)$  represent measurements of displacement as a function of time of the building reacting to different shocks.

$t$	$y_1(t)$	$y_2(t)$
0	0.14	0.50
2	0.38	0.40
4	0.21	-0.12
6	-0.18	-0.49
8	-0.37	-0.33
10	-0.15	0.18
12	0.21	0.48
14	0.34	0.25
16	0.10	-0.23
18	-0.24	-0.45
20	-0.31	-0.18
22	-0.05	0.27

In the course of this project you will address the following questions:

1. Which of the equations

$$\frac{d^2y}{dt^2} + p\frac{dy}{dt} + qy = 0,$$
$$\frac{d^2y}{dt^2} + p\frac{dy}{dt} + qy = ky^3,$$

forms a better model?

2. Having decided which equation is better, determine approximate values of the parameters.
3. Comment on how well the models with your choice of parameters agree with the data. Is there any danger that the building will fall down in a heavy wind storm?

The basic technique is to “guess” values of the constants  $p$ ,  $q$  and  $k$  and to plot the measured data and the solution of the differential equation corresponding to your guess on the same graph. You then change the values of the constants until the computed solution agrees reasonably well with the measured data.

To be successful, you must be systematic. I suggest the following general procedure. Notice that for the building you are studying, the displacements in the  $y_1$  column tend to be smaller than in the  $y_2$  column. We expect that the effect of gravity is minimal for small displacements. Hence, you will:

- (1) Take  $k = 0$  and find values of  $p$  and  $q$  which make the computed solution match the  $y_1$  data as closely as possible.

- (2) *Using the same  $p$  and  $q$*  compare the computed solution to the  $y_2$  data. If the agreement is reasonably close, then leave  $k$  at zero. If the approximation is significantly bad, try increasing  $k$  until a good approximation is obtained.

More specifically, for (1), you will need an initial position and an initial velocity for the building. The given data can be used to find the initial position and the average velocity over the first time interval.

Start off with  $p = 0$  and some (random) value of  $q$ . Plot the solution to the system using this value of  $q$  and the computed initial position and average velocity. You will find that the average velocity does not represent the initial velocity well. Adjust the initial velocity until the solution to the differential equation matches the given data as closely as possible.

Next, note that in a spring,  $q$  represents the stiffness, which controls the rapidity of the vibrations. Adjust the value of  $q$  until the computed period seems to agree with the measured data. Next readjust the initial velocity until the computed curve rises high enough to match the measured data. Finally, adjust  $p$  until you get the amount of damping suggested by the measured data. Make sure that you record the final values of the initial velocity,  $p$  and  $q$ . Then go on to part (2) above.

For your graphs, it is easiest not to use pplane. Instead you will use a MATLAB program “ode23” (which is already in MATLAB). For this, you will need to create a function file which represents the differential equation as a system. Our equation is equivalent with the system

$$\begin{aligned}y' &= v \\v' &= -qy - pv + ky^3\end{aligned}$$

As a matrix equality, this becomes

$$\begin{pmatrix} y \\ v \end{pmatrix}' = \begin{pmatrix} 0 & 1 \\ -q & -p \end{pmatrix} \begin{pmatrix} y \\ v \end{pmatrix} + \begin{pmatrix} 0 \\ ky^3 \end{pmatrix}$$

For MATLAB to deal with this equation, you first must choose values for the constants. Let’s say that you use  $q = 2$ ,  $p = 3$  and  $k = 4$ . Next you pull down the “File” menu and select “New M-File”. You then enter the following statements into this file:

```
function g=yy(t,x)
A=[0,1; -2 -3];
g=A*x+[0;4*x(1)^3];
```

You then select “Save As” and save it as “yy.m”. *Do not close this file.* You will need to modify it every time you change  $p$  and  $q$ . If you have not closed it, you can simply edit it every time these values change and then choose “Save” to inform MATLAB of your changes.

For your plotting, first create a vector  $s$  which represents the time variable. A convenient way of doing this for building 2 is with the command

```
s=0:2:22
```

Next you create vectors `y1` and `y2` which represent the given data. (The syntax would be “`y1=[0.14,0.38 ,..., -0.05]`”). The initial position is 0.14 and the average velocity over the first interval is .12. Next, we write the following commands, just as shown

```
hold on;
cla;
[t,y]= ode23('yy',0,22,[.14;.12]); plot(t,y(:,1)); plot(s,y1);
```

“`cla;`” clears the screen, “`ode23`” solves the system defined by the function “`yy`” over the interval  $0 \leq t \leq 22$  with  $y(0) = .14$  and  $v(0) = .12$ . The first “`plot`” command plots the solution and the second command plots the data in the vector `y1`.

Now, if you need to re-plot your solutions, you simply clear the screen (with “`cla;`”) and then use the up-arrow to return to this line and either edit the file `yy.m` or edit the above line and then hit enter. One final trick: You may want to re-size both the figure window and the command window so that they don’t overlap. This saves jumping back and forth between windows.

For question 3, you might want to enter the system (with the values of  $p$ ,  $q$  and  $k$  you found) into `pplane` and compute a few of the orbits to see if you can make the building collapse with the right initial conditions.

Regarding the question, “How can we tell from our graphs whether the building is going to collapse?” look at the bottom graph on the last page. What is happening to the displacement if the initial position of the building is described by a point on one of the spiral orbits? What if the initial position is described by a point on one of the other orbits?