MA 35100 Assignments

Text: *Linear Algebra, Ideas and Applications, 4rd Edition*, Richard C. Penney, (J. Wiley, 2016).

Typically, each assignment contains exercises from the material discussed in the most recent class as well as material covered earlier. The questions over new material usually consist of a few computational problems. Those over earlier material contain additional computational problems as well as some more conceptual exercises. This has arrangement several advantages. If a student does not get a problem on one assignment, he/she would probably have difficulty with similar problem types. Rather than assigning more problems of the same type on the same assignment, it is better to assign them on several later assignments after the students have had an opportunity to ask questions. This has the effect of increasing the number of assignments in which the student encounters a particular concept without increasing the number of problems the student attempts. It also allows the opportunity to master the basic material before attempting the more advanced material.

Warning: These assignments are only approximate. We may make changes in the assignments as the semester progresses depending on what we actually cover in class.

Assignment 1: Matrices, Linear Independence
Read Pages 1-9, stopping after the statement of Proposition 1.1
Exercises: p.17, 1.1b, 1.3, 1.5a,c,e, 1.7, 1.8, 1.9

Assignment 2: Independence in $\mathbb{R}^2$ and $\mathbb{R}^3$, Span, Vector Space[1]
Read Pages 10-16, 28-30
T-F Questions: p.16, 1.1, 1.3, 1.5, 1.9, 1.12
Exercises: p.17, 1.4, 1.10, 1.12, 1.14, 1.24 a,b,c 1.33 a,b, g, 1.38
p. 38 1.49,1.55b, parts(i) and (ii)

Assignment 3: Simple Systems
Read Pages 28-31, stopping before equation (1.12)
T-F Questions p.16: 1.2, 1.4, 1.6, 1.8
Exercises: p.17: 1.11, 1.15, 1.19b , 1.23, 1.24d, e, 1.33e,f,i
p.38 1.55c,d,h

[1]Note to instructor: The discussion of vector spaces here should be brief. The concept will be developed further over the course of the semester.

Assignment 4: Parametric Representation, Rank[2]
Read Pages 31-38
Exercises: p.17, 1.16, 1.19c , 1.28
p.38: 1.55j, 1.56j (*Hint:* Substitute two different choices of the parameters into the solution from the back of the book.), 1.57 Rank 2 case
p.63, 1.63, 1.64c,d (Bring the matrices into **Reduced Echelon form**),

Assignment 5: Gaussian Elimination
Read Pages 47-58, stopping before Spanning in Polynomial Spaces
T-F Questions: p.38, 1.13, 1.14, 1.15, 1.16, 1.17, 1.18, 1.19
Exercises: p.63, 1.65a,c,g, 1.66a,c,g, (You may skip writing the system down), 1.67a (See Example 1.1 on p.57.), 1.78a, 1.84
p.86, 1.93a,b, 1.99

Assignment 6: Column Space, Subspace, and Linearity
Read Pages 74-79
Exercises: p.63, 1.65b,d,h, 1.66b,d,h, 1.67c, 1.68c, 1.78b, 1.87 (*Hint:* The system is equivalent with the system below.) Also in 1.87, write the general solution using z and w as free variables. The point here is that there are many different, but equivalent, ways of describing the solution. (*Ans:* $[x, y, z, w]^t = z[-1, -2, 1, 0]^t + w[-1, -1, 0, 1]^t$.)

$$\begin{aligned} z + w &= -x \\ 2z + w &= -y \end{aligned}$$

Remark. **Beginning wih Assignment 7 you may use technology to do any row reductions. There are many free online sites that reduce matrices. In general, within a given chapter, you may use technology to do computations taught in preceding chapters but should do computations taught in the current chapter by hand.**

Assignment 7: Nullspace, Translation Theorem
Read p.80-85
Exercises: p.63, 1.79, 1.80, 1.82 (Your "proof" will really be an explanation.)
p.86, 1.100, 1.108, 1.112, 1.114c, 1.116 p.108, 2.1a,g (Test the matrices for independence and, if dependent, **use your answer** to express one as a linear combination of the others as in Examples 2.2 and 2.3. Ignore the part asking for a basis.)

Assignment 8: The Test for Linear Independence
Read p.97-104, stopping before Bases for the Column Space.

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[2]Note: The comment that "we give a better definition of rank in the next section" made on p.36 is incorrect. The reference should be to Definition 2.6 on p.136.

T-F Questions p.85: 1.31, 1.32, 1.33, 1.34, 1.36
Exercises: p.86, 1.101, 1.111, 1.114e, 1.119
p.108, 2.1g (Test the matrices for independence and, if dependent, **use your answer** to express one as a linear combination of the others as in Examples 2.2 and 2.3. Ignore the part asking for a basis.)

Assignment 9: Bases for the Column Space and Dimension

Read p.104-106, stopping before Testing Functions for Independence, and p.114-118
T-F Questions p.85: 1.27, 1.29 *Hint:* The answer is "True." Why?
Exercises: p.86, 1.120, 1.121
p.108, 2.1e,i, 2.3a, 2.12
p.123, 2.33b (*Hint:* Begin by expressing $\mathcal{W}$ as a span. It is not necessary to use the subspace properties in this problem.), 2.34b, 2.37b, 2.38b

Assignment 10: Dimension and Bases for the Row Space

Read p.119-123 and p.132-136
T-F Questions p.108, 2.2b,e, 2.4, 2.7
Exercises:
p.108 2.3c,e,d
p.122, 2.33a
p.143: 2.64, (*Hint:* Do Exercise 2.65 first. The non-zero rows of the row reduced echelon form are a basis of the row space. Attempt to express X and Y as linear combinations of these rows), 2.65, 2.66a, 2.67a

Assignment 11: Rank-Nullity Theorem

Read p.137-142
T-F Questions p.108 2.3, p.122, 2.10,2.11,2.13
Exercises: p.123, 2.31, 2.35d, 2.39 (*Hint:* Use Theorem 2.8, p.119), 2.43
p.143 2.66d, 2.67d, 2.74, rows 1-4 (*Hint:* For an $m \times n$ matrix A "Always Solvable" means that the dimension of the column space is m and "Unique Solution" means that the null space is $\{\mathbf{0}\}$), 2.76a,b,c (*Hint:* Look at the columns of A. For c, think about the dimension of the column space.) **NO ROW REDUCTION IS ALLOWED IN THIS PROBLEM**
p.157, 3.1b, 3.2b,

Assignment 12: Linear Transformations

Read p. 149-156
T-F Questions p.108 2.8, p.122, 2.14,2.18
Exercises:
p.123 2.51, p. 143, 2.74, rows 5-8, 2.78 (**NO ROW REDUCTION IS ALLOWED IN THIS PROBLEM**),2.80

p.159 3.5, 3.10, 3.15a,b, 3.16, 3.17

Assignment 13: Matrix Multiplication
Read p.165-168
Exercises
p.159, 3.1a,c, 3.2a,c 3.4, 3.6, 3.11, 3.19a,b,c, 3.21
p.173, 3.26a, 3.27a,b,c, 3.35, 3.40

Assignment 14: Inverses
Read p.182-188, stopping before "Computational Issues..."
Exercises:
p.173, 3.38, 3.41, 3.46,
p.190 3.63a, 3.64a,b,c, 3.65a,b, 3.66c

Remark. With the exception of 3.6 and 3.64i, which should be done by hand, you can do these exercises on a computer. For example the "double matrix" on the bottom of page 185 can be entered into a computer as the 3×6 matrix

$$D = \begin{bmatrix} 1 & 2 & 1 & 1 & 0 & 0 \\ 1 & 3 & 4 & 0 & 1 & 0 \\ 2 & 4 & 1 & 0 & 0 & 1 \end{bmatrix}$$

Then ask the computer to compute the row reduced form of D. The answer will be the last double matrix on p.193. **You are of course not allowed to do these exercises using the inverse command.**

Assignment 15: Determinants
Read p.238-246, stopping before Section 4.1.1
Exercises:
p.173, 3.47, 3.48, 3.51, 3.59
p.190, 3.64g,h,i, 3.75, 3.76, 3.77, 3.81 (*Hint* 3.77 and 3.81 both rely on Theorem 3.10), 3.85
p.249 4.1a, c, i, 4.2c, 4.4

Assignment 16: Reduction and Determinants (Due 3/23)
Read p.252-257, stopping after the statement of Theorem 4.11
Exercises:
p.190, 3.74, 3.78, 3.84
p.249 4.5
p.258, 4.12a,b,c, 4.15a, 4.17

Assignment 17: Cramer's Rule and a Formula for Inverses, due 3/30
Read p.264-268
Exercises:

p.258, 4.23, 4.24, 4.25,
p.268, In in all of these problems express the answer as a ratio of determinants. **Do Not Compute The Determinants.** 4.34, 4.35, 4.36, 4.37

Assignment 18: Eigenvectors[3]
Read p.271-279
Exercises:
p.279, 5.1a, c (*Hint:* If $AX = \lambda X$ then X is an eigenvector with eigenvalue λ.), 5.3b 5.4b (*Hint:* Linear combinations of eigenvectors corresponding to the **same** eigenvalue are eigenvectors.), 5.5a (See Definition 5.2 on p.278.)

Assignment 19: Diagonalization and Complex Eignevalues
Read p. 287-289, p.296-303, stopping before Complex Vector Spaces
Exercises:
p.279, 5.1b, 5.3c, 5.4c, 5.5e,i (*Hint:* In i the characteristic polynominal is $p(\lambda) = -(\lambda-1)(\lambda-2)^2$), 5.8
p.290, 5.28a, 5.29b
p.304, 5.45 (Do not compute A^{20}. Instead find complex matrices Q and D such that $A = QDQ^{-1}$)

Assignment 20: Coordinates in $\mathbb{R}^n$
Read pages 215-220, stopping before the paragraph before Theorem 3.14.
Exercises:
p.230, 3.117b,c
p.279, 5.12, 5.14
p.290, 5.28b, 5.29d,f, 5.37
p.304, 5.46 (Hint: $p_A(\lambda) = -(\lambda-7)(\lambda^2 - 13\lambda + 49)$), 5.47

Assignment 21: Coordinates for General Vector Spaces
Read pages 220-229, stopping before Invertible Linear Transformations
Exercises:
p.230, 3.117d, 3.118b,c
p.279, 5.13
p.290, 5.33,5.38a, 3
p.304, 5.49

[3]Note to instructor: We suggest that you begin with Definition 5.1 and Example 5.2 and assign Example 5.1 as outside reading.

Assignment 22: Orthogonality in $\mathbb{R}^n$

Read p.308-312, stopping before Orthogonal/Orthonormal Bases and Coordinates

Exercises:

p.230, 3.130c,e(See Example 3.20, p.224) 3.131c, 3.133c(Use formula (3.40) on p. 227)

p.290, 5.34, 5.38b,

p.316, 6.1a,c, 6.2a,c,d, (See Definitions 6.1 and 6.3 on p. 310 and 3.11)

Assignment 23: Orthogonal Bases and Projections

Read p.312-316, p, 319-322, stopping after Example 6.7

Exercises:

p.230, 3.120c, 3.134c

p.316, 6.3,6.7, 6.9 (Use Theorem 6.4 on p.213), 6.12

Assignment 24: The Gram-Schmidt Process

Read p.322-323, stopping before The QR-Decomposition

Exercises:

p.316, 6.13, 6.14, 6.17

p.328, 6.19, 6.20a,b,c,d, 6.21a,b,c,d, 6.22a,c, 6.29