16)

e^{2t-it}

$e^{2t}\cdot[\cos t - i\sin t]$

5 pts.

(1) The following vectors X_1 and Y_1 are eigenvectors for a certain 3×3 matrix A corresponding to the eigenvalues $2-i$ and -4 respectively. Find the general solution to the system $X' = AX$ **in real form**. No complex numbers allowed!

$$x_1 = \begin{bmatrix} i+1 \\ i \\ -2i \end{bmatrix}, \quad y_1 = \begin{bmatrix} 1 \\ 3 \\ -1 \end{bmatrix}.$$

For eigenvalues $r = 2-i$:

$$X = e^{rt}x_1$$

$$= e^{(2-i)t}\begin{bmatrix} i+1 \\ i \\ -2i \end{bmatrix}$$

$$= \begin{bmatrix} i+1 \\ i \\ -2i \end{bmatrix}[e^{2t}\cos t - ie^{2t}\sin t]$$

$$= \begin{bmatrix} e^{2t}\sin t + e^{2t}\cos t + ie^{2t}\cos t - ie^{2t}\sin t \\ e^{2t}\sin t + ie^{2t}\cos t \\ -2e^{2t}\sin t - 2ie^{2t}\cos t \end{bmatrix}$$

$$= \begin{bmatrix} \sin t + \cos t \\ \sin t \\ -2\sin t \end{bmatrix}e^{2t} + \begin{bmatrix} \cos t - \sin t \\ \cos t \\ -2\cos t \end{bmatrix}ie^{2t}$$

$\therefore \left\{ \begin{bmatrix} \sin t + \cos t \\ \sin t \\ -2\sin t \end{bmatrix}e^{2t}, \begin{bmatrix} \cos t - \sin t \\ \cos t \\ -2\cos t \end{bmatrix}e^{2t} \right\}$ is the fundamental set of $r = 2-i$.

For eigenvalues $r = -4$:

$$Y = e^{-4t}\begin{bmatrix} 1 \\ 3 \\ -1 \end{bmatrix}$$

$\therefore$ the general solution is

$$X(t) = C_1e^{2t}\begin{bmatrix} \sin t + \cos t \\ \sin t \\ -2\sin t \end{bmatrix} + C_2e^{2t}\begin{bmatrix} \cos t - \sin t \\ \cos t \\ -2\cos t \end{bmatrix} + C_3e^{-4t}\begin{bmatrix} 1 \\ 3 \\ -1 \end{bmatrix}$$

100

(2) The characteristic polynomial for the following matrix is $p(r) = -(r-4)^2(r-5)$ and the vector Y_1 is an eigenvector corresponding to $r = 5$. Find the general solution to the system $X' = AX$. 10 pts.

$$\begin{bmatrix} 7 & 0 & 1 \\ -4 & 2 & 0 \\ -6 & -3 & 4 \end{bmatrix} \quad Y_1 = \begin{bmatrix} -3 \\ 4 \\ 6 \end{bmatrix}$$

For $r=4$: $(r-4)$ is a nil potent,

to find the generalized eigenvectors ; by solving

$$(A-4I)^2 X = 0$$

let $B = A - 4I = \begin{bmatrix} 3 & 0 & 1 \\ -4 & -2 & 0 \\ -6 & -3 & 0 \end{bmatrix}$

$$B^2 = \begin{bmatrix} 3 & 0 & 1 \\ -4 & -2 & 0 \\ -6 & -3 & 0 \end{bmatrix}\begin{bmatrix} 3 & 0 & 1 \\ -4 & -2 & 0 \\ -6 & -3 & 0 \end{bmatrix} = \begin{bmatrix} 3 & -3 & 3 \\ -4 & 4 & -4 \\ -6 & 6 & -6 \end{bmatrix}$$

$$(A-4I)^2 X = 0$$

$$\Rightarrow \quad B^2 X = 0$$

$$\begin{bmatrix} 3 & -3 & 3 \\ -4 & 4 & -4 \\ -6 & 6 & -6 \end{bmatrix} X = 0$$

$$\left[\begin{array}{ccc|c} 3 & -3 & 3 & 0 \\ -4 & 4 & -4 & 0 \\ -6 & 6 & -6 & 0 \end{array}\right] \xrightarrow[\frac{R_2}{4},\ \frac{R_3}{6}]{\frac{R_1}{3}} \left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ -1 & 1 & -1 & 0 \\ -1 & 1 & -1 & 0 \end{array}\right]$$

$$\xrightarrow[R_1+R_2]{R_1+R_3} \left[\begin{array}{ccc|c} 1 & -1 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$X = [x, y, z]$, $x - y + z = 0$

let $z = s$, $y = t$, $x = t - s$

$$\therefore \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} t-s \\ t \\ s \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} t + \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} s$$

$$\therefore X_{(1)} = e^{tA}\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} = e^{(B+4I)t}\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} = e^{4t} \cdot e^{tB}\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} = e^{4t}\left[I + tB + \frac{(tB)^2}{2!} + \cdots\right]\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$$

Since $B^2 X = 0$,

$$= e^{4t}[I + tB]\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$$

$$= e^{4t}\left[\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + t\begin{bmatrix} 3 \\ -6 \\ -9 \end{bmatrix}\right]$$

$$X_{(2)} = e^{tA}\begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} = e^{4t} \cdot e^{tB}\begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} = e^{4t}[I + tB]\begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} = e^{4t}\left[\begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} + t\begin{bmatrix} -2 \\ 4 \\ 6 \end{bmatrix}\right]$$

For $r = 5$,

$$y = e^{5t}\begin{bmatrix} -3 \\ 4 \\ 6 \end{bmatrix}$$

$\therefore$ the general solution is $X(t) = c_1 e^{4t}\begin{bmatrix} 1+3t \\ 1-6t \\ -9t \end{bmatrix} + c_2 e^{4t}\begin{bmatrix} -1-2t \\ 4t \\ 1+6t \end{bmatrix} + c_3 e^{5t}\begin{bmatrix} -3 \\ 4 \\ 6 \end{bmatrix}$

/100

5 pts.

(3) Given that X_1, X_2 and X_3 are eigenvectors for the following matrix, find the general solution to $X' = AX$. *Hint:* To find the eigenvalue, compute AX_i.

$$A = \begin{bmatrix} 2 & 6 & -3 \\ -2 & -10 & 5 \\ -6 & -12 & 5 \end{bmatrix} \quad X_1 = \begin{bmatrix} 1 \\ -1 \\ -2 \end{bmatrix} \quad X_2 = \begin{bmatrix} -1 \\ 2 \\ 2 \end{bmatrix} \quad X_3 = \begin{bmatrix} -1 \\ 3 \\ 5 \end{bmatrix}$$

To find r_1 :

$$(A - r_1 I)X_1 = 0$$

$$\begin{bmatrix} 2-r & 6 & -3 \\ -2 & -10-r & 5 \\ -6 & -12 & 5-r \end{bmatrix}\begin{bmatrix} 1 \\ -1 \\ -2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \qquad \therefore X_{(1)} = e^{2t}\begin{bmatrix} 1 \\ -1 \\ -2 \end{bmatrix}$$

$$(2-r) - 6 + 6 = 0$$

$$r = 2$$

To find r_2 :

$$(A - r_2 I)X_2 = 0$$

$$\begin{bmatrix} 2-r & 6 & -3 \\ -2 & -10-r & 5 \\ -6 & -12 & 5-r \end{bmatrix}\begin{bmatrix} -1 \\ 2 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$-(2-r) + 12 - 6 = 0 \qquad \therefore X_{(2)} = e^{-4t}\begin{bmatrix} -1 \\ 2 \\ 2 \end{bmatrix}$$

$$r = -4$$

To find r_3 :

$$(A - r_3 I)X_3 = 0$$

$$\begin{bmatrix} 2-r & 6 & -3 \\ -2 & -10-r & 5 \\ -6 & -12 & 5-r \end{bmatrix}\begin{bmatrix} -1 \\ 3 \\ 5 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$-2 + r + 18 - 15 = 0 \qquad \therefore X_{(3)} = e^{-t}\begin{bmatrix} -1 \\ 3 \\ 5 \end{bmatrix}$$

$$r = -1$$

∴ the general solution is

$$x(t) = c_1 e^{2t}\begin{bmatrix} 1 \\ -1 \\ -2 \end{bmatrix} + c_2 e^{-4t}\begin{bmatrix} -1 \\ 2 \\ 2 \end{bmatrix} + c_3 e^{-t}\begin{bmatrix} -1 \\ 3 \\ 5 \end{bmatrix}$$

10 0

(4) The following matrix A has characteristic polynomial $p(r) = -(r-5)^3$. 10 pts.

(a) Find e^{tA}.

(b) Find the general solution to $X' = AX$.

$$\begin{bmatrix} 5 & 2 & 1 \\ 0 & 5 & 0 \\ 0 & 1 & 5 \end{bmatrix}$$

(a) $(r-5)$ is a nil potent.

let $B = A - 5I$

$$= \begin{bmatrix} 0 & 2 & 1 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

$$B^2 = \begin{bmatrix} 0 & 2 & 1 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 2 & 1 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$B^3 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\therefore e^{tA} = e^{t(B+5I)} = e^{5t} \cdot e^{tB}$$

$$= e^{5t}\left[I + tB + \frac{(tB)^2}{2!} + \frac{(tB)^3}{3!} + \cdots\right]$$ Since $B^3 = 0$,

$$= e^{5t}\left[I + tB + \frac{t^2}{2}B^2\right]$$

$$= e^{5t}\left[\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + t\begin{bmatrix} 0 & 2 & 1 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} + \frac{t^2}{2}\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\right]$$

$$= e^{5t}\begin{bmatrix} 1 & 2t+\frac{t^2}{2} & t \\ 0 & 1 & 0 \\ 0 & t & 1 \end{bmatrix}$$

(b) $X = e^{tA}X_0.$

$\therefore$ the general solution is

$$X(t) = e^{5t}\begin{bmatrix} 1 & 2t+\frac{t^2}{2} & t \\ 0 & 1 & 0 \\ 0 & t & 1 \end{bmatrix}\begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix}$$

$$= e^{5t}\begin{bmatrix} c_1 + c_2(2t+\frac{t^2}{2}) + c_3 t \\ c_2 \\ c_2 t + c_3 \end{bmatrix}$$

100

(5) A certain 6×6 matrix A has characteristic polynomial $p(r) = -(r-2)^2(r-5)^4$. Let X be a generalized eigenvector for A corresponding to $r = 5$. Give a formula for $e^{tA}X$ that does not require summing an infinite series. Your formula should use as few matrix products as possible relative to the given information.

5 pts.

Let $(A-5I)^4 X = 0$

$B = A - 5I \Rightarrow A = B + 5I$

$e^{tA} = e^{(B+5I)t} = e^{5t} \cdot e^{tB}$

$= e^{5t} \cdot [1 + tB + \frac{(tB)^2}{2!} + \frac{(tB)^3}{3!} + \frac{(tB)^4}{4!} \cdots]$ ✓

Since $B^4 X = 0$.

$\therefore e^{tA}X = e^{5t}[1 + tB + \frac{(tB)^2}{2!} + \frac{(tB)^3}{3!}]X$

(6) Find all singular points for the following differential equation and state which are regular. **Don't forget to justify your answers!**

6 pts.

$$x^2(2x-5)^3y'' + x^3(2x-5)y' + (x-1)y = 0$$

To find the singular points:

$x^2(2x-5)^3 = 0$

$\therefore x=0$ and $x=\frac{5}{2}$

$\therefore x=0$ and $x=\frac{5}{2}$ are the singular points.

Case ①: Assume that $x=0$ is a regular singular point,

$x^2(2x-5)^3y'' + x(x^2(2x-5))y' + (x-1)y = 0$

$p(x) = (2x-5)^3$, $q(x) = x^2(2x-5)$, $r(x)\ (x-1)$

$p(0) = -125$

since $p(x)$, $q(x)$ and $r(x)$ are analytic and $p(0) = -125 \neq 0$, ✓

$\therefore x=0$ is a regular singular point #

Case ②. Assume that $x=\frac{5}{2}$ is a regular singular point.

$[x^2(2x-5)](2x-5)^2y'' + x^3(2x-5)y' + (x-1)y = 0$

$p(x) = x^2(2x-5)$, $q(x) = x^3$, $r(x) = (x-1)$

$p(\frac{5}{2}) = 0$ ✓

Since $p(\frac{5}{2}) = 0$, $\therefore x = \frac{5}{2}$ is not a regular singular point. #

100

(7) Substitute $y = \sum_{n=0}^{\infty} a_n x^n$ into the differential equation

$$4y'' + (x^2 + 4)y' + y = 0$$

and simplify until you obtain an expression of the form

$$\sum_{n=?}^{\infty} ?x^n + \sum_{n=?}^{\infty} ?x^n + \sum_{n=?}^{\infty} ?x^n = 0$$

where the exponent of x in each sum is n and the question marks are explicit expressions. (You do not need to use exactly 3 summation signs.) **Do not simplify further!** 10 pts.

$y = \sum_{n=0}^{\infty} a_n x^n$, $y' = \sum_{n=0}^{\infty} n a_n x^{n-1}$, $y'' = \sum_{n=0}^{\infty} n(n-1) a_n x^{n-2}$

For $y'' \Rightarrow$ $4y'' = \sum_{n=0}^{\infty} 4n(n-1) a_n x^{n-2}$

let $m = n-2$, $n = m+2$

$$4y'' = \sum_{m=-2}^{\infty} 4(m+2)(m+1) a_{m+2} x^m$$

$$4y'' = \sum_{n=-2}^{\infty} 4(n+1)(n+2) a_{n+2} x^n$$

For $y' \Rightarrow$ $x^2 y' = \sum_{n=0}^{\infty} n a_n x^{n+1}$

let $m = n+1$, $n = m-1$

$$x^2 y' = \sum_{m=1}^{\infty} (m-1) a_{m-1} x^m$$

$$x^2 y' = \sum_{m=1}^{\infty} (n-1) a_{n-1} x^n$$

$$4y' = \sum_{n=0}^{\infty} 4n a_n x^{n-1}$$

let $m = n-1$, $n = m+1$

$$4y' = \sum_{m=-1}^{\infty} 4(m+1) a_{m+1} x^m$$

$$4y' = \sum_{n=-1}^{\infty} 4(n+1) a_{n+1} x^n$$

$$\therefore \sum_{n=-2}^{\infty} 4(n+1)(n+2) a_{n+2} x^n + \sum_{n=1}^{\infty} (n-1) a_{n-1} x^n + \sum_{n=-1}^{\infty} 4(n+1) a_{n+1} x^n + \sum_{n=0}^{\infty} a_n x^n = 0$$

100

10 pts.

(8) In attempting to solve a certain differential equation, we substituted $y = \sum_{n=0}^{\infty} a_n x^n$ into the differential equation and simplified, obtaining

$$\sum_{n=0}^{\infty} 3n(n-1)a_n x^{n-2} + \sum_{n=0}^{\infty} -3n(n-1)a_n x^n + \sum_{n=0}^{\infty} -3na_n x^n + \sum_{n=0}^{\infty} -a_n x^n = 0.$$

(a) Continue the solution process to obtain the recursion relation.

$$\sum_{n=0}^{\infty} 3n(n-1)\, a_n x^{n-2}$$

let $m = n-2$, $n = m+2$

$$\sum_{n=0}^{\infty} 3n(n-1)\, a_n x^{n-2} = \sum_{m=-2}^{\infty} 3(m+2)(m+1)\, a_{m+2} x^m$$

$$= \sum_{n=-2}^{\infty} 3(n+1)(n+2)\, a_{n+2} x^n$$

$$= \sum_{n=0}^{\infty} 3(n+1)(n+2)\, a_{n+2} x^n$$

$$\sum_{n=0}^{\infty} \left[3(n+1)(n+2)a_{n+2} - 3n(n-1)a_n - 3na_n - a_n\right] x^n = 0$$

$\therefore$ the recursion relationship is

$$3(n+1)(n+2)\, a_{n+2} - 3n(n-1)\, a_n - 3n a_n - a_n = 0$$

$$a_{n+2} = \frac{(3n(n-1) + 3n + 1)\, a_n}{3(n+1)(n+2)}$$

$$= \frac{(3n^2+1)}{3(n+1)(n+2)}\, a_n \quad , n = 0, 1, 2, \ldots$$

(b) Find the first three **non-zero** terms of the power series expansion for the solution y_1 satisfying $y_1(0) = 0, y_1'(0) = 2$.

$$y = \sum_{n=0}^{\infty} a_n x^n = a_0 + a_1 x^1 + a_2 x^2 + \cdots$$

$y_1(0) = 0$, $a_0 = 0$

$y_1'(0) = 2$, $a_1 = 2$

Since $a_0 = 0$, $a_2, a_4, a_6 \ldots = 0$

$n = 1$, $a_3 = \frac{4}{3\cdot 2\cdot 3} a_1 = \frac{4}{9}$

$n = 3$, $a_5 = \frac{28}{3\cdot 4\cdot 5} a_3 = \frac{28}{3\cdot 4\cdot 5}\left[\frac{4}{9}\right] = \frac{28}{3\cdot 5\cdot 9}$

$$\therefore y_1 = \sum_{n=0}^{\infty} a_n x^n = a_1 x + a_3 x^3 + a_5 x^5$$

$$= 2x + \frac{4}{9}x^3 + \frac{28}{3\cdot 5\cdot 9} x^5$$

99

(9) The following differential equation has a regular singularity at $x = 0$. 9 pts.

$$x^2(x^3 + 2x + 1)y'' + x(4x^3 + x + 6)y' + (x + 6)y = 0.$$

(a) Give the approximating Euler equation.

$p(x) = x^3 + 2x + 1$, $p(0) = 1$

$q(x) = 4x^3 + x + 6$, $q(0) = 6$

$r(x) = x + 6$, $r(0) = 6$

$\therefore$ the approximating Euler equation is

$$x^2 y'' + 6xy' + 6y = 0$$

(b) Give the indicial equation.

let $y = x^r$, $y' = r x^{r-1}$, $y'' = r(r-1) x^{r-2}$

$r(r-1)x^r + 6rx^r + 6x^r = 0$

$(r^2 - r + 6r + 6)x^r = 0$

$(r^2 + 5r + 6) = 0$

$\therefore$ the indicial equation is $r^2 + 5r + 6 = 0$

(c) Use Theorem 5.6.1 on p. 289 of the text to describe the expected form of the solutions. **Do not find the coefficients of the series expansions!**

$r^2 + 5r + 6 = 0$

$(r+3)(r+2) = 0$

$r_1 = -2$ and $r_2 = -3$

not

Since $r_1 > r_2$, $\therefore y_1(x) = x^{-2} \sum_{n=0}^{\infty} a_n x^n$ −1

Since $r_1 - r_2 = -2 - (-3) = 1$ = +ve integer, Not

$\therefore y_2(x) = a y_1(x) \ln|x| + x^{-3} \sum_{n=0}^{\infty} b_n x^n$

99

6 pts.

(10) You are given that $y(x) = x^2$ is a solution of the following differential equation. Use the method of reduction of order to find a second independent solution. **Other methods will not receive credit!**

$$x^2y'' - 3xy' + 4y = 0.$$

Using method of reduction of order,

$$y'' + p(x)y' + q(x)y = 0$$

$$y_2 = u(x)y_1$$

$$= u(x)x^2 \quad , \text{ where } u' = (y_1(x))^{-2}e^{-\int p(x)dx}$$

$$p(x) = -\frac{3x}{x^2} = -\frac{3}{x}$$

$$u' = x^{-4}e^{-\int -\frac{3}{x}\,dx}$$

$$= x^{-4}e^{\ln|x|^3}$$

$$= x^{-4}x^3$$

$$u' = x^{-1}$$

$$u = \int x^{-1}\,dx$$

$$= \ln|x|$$

$$\therefore \quad y_2 = \ln|x|\,x^2$$

$$= x^2\ln|x|$$

99

(11) *Use the definition of the Laplace transform* to compute the Laplace transform $\mathcal{L}(f)$ of the function $f(t)$ defined below. **Other methods will not receive credit!** 4 pts.

$$f(t) = \begin{cases} e^{2t}, & 0 \le t < 3 \\ 4, & 3 \le t. \end{cases}$$

By the definition of the Laplace transform,

$$\mathcal{L}(f)(s) = \int_0^\infty f(t) e^{-st}\, dt$$

$$= \int_0^3 e^{2t} e^{-st}\, dt + \int_3^\infty 4 e^{-st}\, dt$$

$$= \int_0^3 e^{(2-s)t} \frac{d((2-s)t)}{(2-s)} + \int_3^\infty 4 e^{-st} \frac{d(-st)}{-s}$$

$$= \frac{e^{(2-s)t}}{(2-s)}\Bigg|_0^3 + \left(\frac{4e^{-st}}{-s}\Bigg|_3^\infty \right)$$

$$= \frac{e^{(2-s)(3)}}{2-s} - \frac{1}{2-s} + \frac{4e^{-3s}}{s}$$

$$= \frac{1 - e^{(6-3s)}}{(s-2)} + \frac{4e^{-3s}}{s}$$

49

4 pts. (12) Find the inverse Laplace transform of

$$F(s) = \frac{2s^2+4}{s(s^2+4)}.$$

By partial fraction, $\frac{2s^2+4}{s(s^2+4)} \equiv \frac{A}{s} + \frac{Bs+C}{s^2+4}$

$2s^2+4 \equiv A(s^2+4) + (Bs+C)s$

$s=0$, $4A=4$, $A=1$, $A+B=2$, $B=2-1=1$ $C=0$

$F(s) = \frac{1}{s} + \frac{s}{s^2+4}$

$F(s) = \mathcal{L}(1+\cos 2t)$ ✓

$\therefore f(t) = 1 + \cos 2t$

4 pts. (13) Find the inverse Laplace transform of

$$F(s) = \frac{e^{-5s}(2s^2+4)}{s(s^2+4)}.$$

Let $G(s) = \frac{2s^2+4}{s(s^2+4)}$

$= \frac{1}{s} + \frac{s}{s^2+4}$

$\therefore g(t) = 1 + \cos 2t = f(t)$.

$F(s) = e^{-5s} f(t) = \mathcal{L}(u_5(t) f(t-5))$

$= \mathcal{L}(u_5(t)(1+\cos(2(t-5))))$ ✓

$\therefore f(t) = u_5(t)(1+\cos(2t-10))$

99

(14) Find the inverse Laplace transform of 4 pts.

$$F(s) = \frac{e^{2s}}{(s+2)^5}.$$

Let $G(s) \quad \frac{1}{(s+2)^5}$

$$= \frac{1}{4!} \frac{4!}{(s+2)^5} = F(s+2)$$

$$h(s) = \frac{4!}{s^5} = t^4$$

$$\therefore G(s) = F(s+2) = \mathcal{L}\left(\tfrac{1}{24} e^{-2t} f(t)\right) = \mathcal{L}\left(\tfrac{1}{24} e^{-2t} t^4\right)$$

$$\therefore g(t) = \frac{1}{24} e^{-2t} t^4$$

$$F(s) = e^{2s} G(s) = \mathcal{L}\left(u_{(-2)}(t)\, g(t+2) \right)$$

$$= \mathcal{L}\left(u_{(-2)}(t) \left(e^{-2(t+2)} (t+2)^4 \left(\tfrac{1}{24}\right)\right)\right)$$

✓

$$\therefore f(t) = \frac{1}{24} u_{(-2)}(t)\, e^{-2(t+2)} (t+2)^4 \quad \#$$