~num ...

1. a) $u = \frac{58.54}{50} = 1.1708$

b) $d = \frac{43.86}{50} = .8772$

c) $u \cdot d = e^{(r-\delta)h+\sigma\sqrt{h}} \, e^{(r-\delta)h-\sigma\sqrt{h}} = e^{2(r-\delta)h} = e^{2(r)(\frac{1}{3})} = (1.1708)(.8772)$

$\Rightarrow \quad r = .0400$

2. a) $p^* = \dfrac{e^{(r-\delta)h} - d}{u - d} = \dfrac{e^{(.08-.3357)(.25)} - .8}{1.1 - .8} = .4603$

b) $C = e^{-.08(.75)}\left[\binom{3}{0}(p^*)^3(q^*)^0(53.1) + \binom{3}{1}(p^*)^2(q^*)^1(16.8)\right] = 10.3025$

c) $\Delta(96.8)e^{.3357(.25)} + Be^{.08(.25)} = 16.8$

$\Delta(70.4)e^{.3357(.25)} + Be^{.08(.25)} = 0$

$28.7112\,\Delta = 16.8 \quad \Rightarrow \quad \Delta = .5851 \quad \Rightarrow \Delta S = 51.4920$

$\Rightarrow \quad B = -43.9129$ (i.e. we borrow \$43.9129)

d) Early exercise value $= 88 - 80 = 8$, value of call if held $= \Delta S + B = 7.5791$

$\Rightarrow$ Early exercise would occur

3. a) $F^P_{0,T} = 100 - 5e^{-.08(.5)} = 95.1961 \Rightarrow \sigma_F = \sigma_S \cdot \frac{S}{F^P_{0,T}} = .2\,\frac{100}{95.1961} = .2101$

b) $u = e^{(.08)(.25) + .2101\sqrt{.25}} = 1.1332$

c) $d = e^{(.08)(.25) - .2101\sqrt{.25}} = .9185$

d) $F^P_{0,T} = 95.1961$

e) $F_u = 95.1961\,(1.1332) = 107.8767$

f) $S_u = F_u + PV(div) = 107.8767 + 5e^{-.08(.25)} = 112.7777$

4. Convexity requirement: $\frac{20-x}{50-30} \geq \frac{x-5}{60-50} \Leftrightarrow \frac{20-x}{20} \geq \frac{x-5}{10}$

$\Rightarrow 200 - 10x \geq 20x - 100 \Leftrightarrow 300 \geq 30x \Leftrightarrow 10 \geq x$

Thus, an arbitrage opportunity exists if $x > 10$

5. Convexity violated? $\frac{35-10}{50-30} \overset{?}{\leq} \frac{40-35}{60-50} \Rightarrow 1.25 \overset{?}{\leq} .5$ Convexity is violated

$50 = 30\lambda + 60(1-\lambda) \Rightarrow \lambda = \frac{60-50}{60-30} = \frac{1}{3}$

Thus the arbitrage is to buy 1 30-strike put, buy 2 60-strike puts, and sell 3 50-strike puts.

6. No-arbitrage requirement : $x - 25 \le 50 - 20 \iff x \le 55$

Thus, there is an arbitrage opportunity if $x > 55$. In such a case, the arbitrage would be to buy the 20-strike put and sell the 50-strike put.

7. $r_{\$} = .04$, $r_{Y} = .02$, $x_0 = 101\ Y/\$$

$-denominated call : right to exchange \$.80 for 100 Y at $t = \frac{1}{12}$ (price is \$

Y-denominated put : right to exchange \$1 for 125 Y at $t = \frac{1}{12}$ (price is

$\Rightarrow \quad Z = (1.25)(101)x = 126.25\, x$ yen

8. $\sigma_{\$} = .12$. $u = e^{(.02-.04)(\frac{1}{12})(\frac{1}{4}) + .12\sqrt{(\frac{1}{12})(\frac{1}{4})}} = 1.0170$

9. $C - P = [S - PV(div)] - PV(K) \Rightarrow 6.50 - P = 74.20 - 1.1e^{-.06(\frac{1}{4})} - 1.1e^{-.06(\frac{1}{2})} - 70e^{-.06}$

$\Rightarrow \quad P = 2.3823$, the theoretical price of the put

Thus, the arbitrage is to sell the put for 2.50 and synthetically create the put, which costs 2.3823 for a risk-free profit of 0.1177

10. $.114 - .098 = x_0 e^{-.06(.5)} - 0.94 e^{-.07(.5)} \Rightarrow x_0 = \$0.9518 / €$