```
> ud:=proc(r,h,del,sig) global u,d;
  u:=evalf(exp((r-del)*h+sig*sqrt(h)));
  d:=evalf(exp((r-del)*h-sig*sqrt(h))):
  end proc;
```

$ud := \mathbf{proc}(r, h, del, sig)$ **(1)**

global u, d;

$u := evalf(\exp((r - del) * h + sig * \mathrm{sqrt}(h)))$; $d := evalf(\exp((r - del) * h - sig * \mathrm{sqrt}(h)))$

end proc

```
> pq:=(r,h,del,u,d)->(exp((r-del)*h)-d)/(u-d);
```

$$pq := (r, h, del, u, d) \rightarrow \frac{e^{(r - del)\,h} - d}{u - d} \quad \textbf{(2)}$$

```
> S:=50;K:=45;sig:=.1;r:=.05;h:=.25;del:=.01;
```

$S := 50$

$K := 45$

$sig := 0.1$

$r := 0.05$

$h := 0.25$

$del := 0.01$ **(3)**

```
> Tree:=proc(u,d,S) global C0,C00,C01,C02,C12,C11,C10,C23,C22,C21,
  C20,C34,C33,C32,C31,C30;
  C01:=u*S;C00:=d*S;C12:=u^2*S;C11:=u*d*S;C10:=d^2*S;
  C23:=u^3*S;C22:=u^2*d*S;C21:=u*d^2*S;C20:=d^3*S;
  C34:=u^4*S;C33:=u^3*d*S;C32:=u^2*d^2*S;C31:=u*d^3*S;C30:=d^4*S;
  end proc;
```

$Tree := \mathbf{proc}(u, d, S)$ **(4)**

global $C0, C00, C01, C02, C12, C11, C10, C23, C22, C21, C20, C34, C33, C32, C31, C30$;

$C01 := u * S$;

$C00 := d * S$;

$C12 := u\text{^}2 * S$;

$C11 := u * d * S$;

$C10 := d\text{^}2 * S$;

$C23 := u\text{^}3 * S$;

$C22 := u\text{^}2 * d * S$;

$C21 := u * d\text{^}2 * S$;

$C20 := d\text{^}3 * S$;

$C34 := u\text{^}4 * S$;

$C33 := u\text{^}3 * d * S$;

$C32 := u\text{^}2 * d\text{^}2 * S$;

$C31 := u * d\text{^}3 * S$;

$C30 := d\text{^}4 * S$

```
end proc
```

```
> Price:=proc(r,h,del,Cu,Cd,S) global SDel,p,q,C,B,Del;
  SDel:=exp(-del*h)*(Cu-Cd)/(u-d);
  p:=(exp((r-del)*h)-d)/(u-d);q:=1-p;
  C:=exp(-h*r)*(p*Cu+q*Cd);
  B:=C-SDel:
  Del:=SDel/S:
  end proc;
```

$$Price := \mathbf{proc}(r, h, del, Cu, Cd, S) \quad \textbf{(5)}$$

```
    global SDel, p, q, C, B, Del;
    SDel := exp(-h*del)*(Cu-Cd)/(u-d);
    p := (exp((r-del)*h)-d)/(u-d);
    q := 1-p;
    C := exp(-h*r)*(p*Cu+q*Cd);
    B := C-SDel;
    Del := SDel/S
end proc
```

```
> POC:=(S,K)->max(S-K,0);POP:=(S,K)->max(K-S,0);
```

$$POC := (S, K) \rightarrow \max(S - K, 0)$$
$$POP := (S, K) \rightarrow \max(K - S, 0) \quad \textbf{(6)}$$

Problem 1.

```
> S:=100; K:=90;r:=.05;u:=1.1;d:=.8;h:=.25;del:=0;
```

$$S := 100$$
$$K := 90$$
$$r := 0.05$$
$$u := 1.1$$
$$d := 0.8$$
$$h := 0.25$$
$$del := 0 \quad \textbf{(7)}$$

```
> Tree(u,d,S):
```

```
> C01;C00;
```

$$110.0$$
$$80.0 \quad \textbf{(8)}$$

```
> C12;C11;C10;
```

$$121.00$$
$$88.00$$
$$64.00 \quad \textbf{(9)}$$

```
> UU:=POP(C12,K);UD:=POP(C11,K);DD:=POP(C10,K);
```

$$UU := 0$$
$$UD := 2.00$$
$$DD := 26.00 \quad \textbf{(10)}$$

```
> p:=pq(r,h,del,u,d);q:=1-p;
```

$$p := 0.7085948400$$
$$q := 0.2914051600 \quad \textbf{(11)}$$

Note that r is an annual rate so we discount back r/2

```
> PR3:=exp(-r/2)*(UU*p^2+2*UD*p*q+DD*q^2);
```

$$PR3 := 2.958889281 \quad \textbf{(12)}$$

Part (d)

```
> Price(r,h,del,UU,UD,C01):
> C;B;SDel;Del;
```

$$0.5755705339$$
$$7.242237201$$
$$-6.666666667$$
$$-0.06060606061 \quad \textbf{(13)}$$

```
> CPU:=C;
```

$$CPU := 0.5755705339 \quad \textbf{(14)}$$

Part(e)

```
> Price(r,h,del,UD,DD,C00):
> C;B;SDel;Del;
```

$$8.882002008$$
$$88.88200201$$
$$-80.00000000$$
$$-1.000000000 \quad \textbf{(15)}$$

```
> CPD:=C;
```

$$CPD := 8.882002008 \quad \textbf{(16)}$$

Part (c)

```
> Price(r,h,del,CPU,CPD,S):
> C;B;SDel;Del;
```

$$2.958889281$$
$$30.64699419$$
$$-27.68810491$$
$$-0.2768810491 \quad \textbf{(17)}$$

Problem 2.

```
> S:=50; K:=45;r:=.05;sig:=.1;h:=.25;del:=.01;
```

$$S := 50$$
$$K := 45$$
$$r := 0.05$$
$$sig := 0.1$$
$$h := 0.25$$
$$del := 0.01 \quad \textbf{(18)}$$

2(a)

```
> ud(r,h,del,sig):
> u;d;
```

1.061836547

0.9607894392 **(19)**

2(b)

```
> p:=pq(r,h,del,u,d);q:=1-p;
```

$p := 0.4875026002$

$q := 0.5124973998$ **(20)**

This computes the last row of the tree, which was given

```
> Tree(u,d,S);
```

42.60718946 **(21)**

```
> C34;C33;C32;C31;C30;
```

63.56245760

57.51369005

52.04053880

47.08822670

42.60718946 **(22)**

Pay off for last row:

```
> CP34:=POC(C34,K);CP33:=POC(C33,K);CP32:=POC(C32,K);CP31:=POC
  (C31,K);CP30:=POC(C30,K);
```

$CP34 := 18.56245760$

$CP33 := 12.51369005$

$CP32 := 7.04053880$

$CP31 := 2.08822670$

$CP30 := 0$ **(23)**

Use the binomial coefficients to find price of the call

```
> Ans:=exp(-r)*(CP34*p^4+4*CP33*p^3*q+6*CP32*p^2*q^2+4*CP31*p*q^3)
  ;
```

$Ans := 6.854189495$ **(24)**

Part (d), Value of the option at Su (Binomial coefficients are 1,3,3,1)

```
> exp(-r*(3*h))*(CP34*p^3+3*CP33*p^2*q+3*CP32*p*q^2+CP31*q^3);
```

9.351379338 **(25)**

Part (e)

```
> CP34:=POP(C34,K);CP33:=POP(C33,K);CP32:=POP(C32,K);CP31:=POP
  (C31,K);CP30:=POP(C30,K);
```

$CP34 := 0$

$CP33 := 0$

$CP32 := 0$

$CP31 := 0$

$CP30 := 2.39281054$ **(26)**

```
> Ans:=exp(-r)*(CP30*q^4);
```

$Ans := 0.1570219094$ **(27)**

Part (f) Value at node:

```
> CP34:=POC(C34,K);CP33:=POC(C33,K);
```

$$CP34 := 18.56245760$$
$$CP33 := 12.51369005 \quad \textbf{(28)}$$

```
> exp(-r*h)*(CP34*p+CP33*q);
```

$$15.27040195 \quad \textbf{(29)}$$

Value of exercise: (We do not exercise)

```
> C23;K;C23-K;
```

$$59.86086825$$
$$45$$
$$14.86086825 \quad \textbf{(30)}$$

Part (g)

```
> ud(r,1/20,del,sig):u;d;
```

$$1.024659825$$
$$0.9798451992 \quad \textbf{(31)}$$

```
> Ans:=u^(18)*d^2*S;
```

$$Ans := 74.42514660 \quad \textbf{(32)}$$

Problem 3.

```
> S:=50; K:=45;r:=.05;sig:=.1;h:=.25;del:=0;
```

$$S := 50$$
$$K := 45$$
$$r := 0.05$$
$$sig := 0.1$$
$$h := 0.25$$
$$del := 0 \quad \textbf{(33)}$$

Present value of dividend

```
> PVD:=exp(-r/2)*3;
```

$$PVD := 2.925929736 \quad \textbf{(34)}$$

Part (a) Price of the prepaid forward:

```
> S0:=S-PVD;
```

$$S0 := 47.07407026 \quad \textbf{(35)}$$

Part (b)

```
> sig0:=sig*(S/S0);
```

$$sig0 := 0.1062155869 \quad \textbf{(36)}$$

Part (c) Value of tree at node:

```
> Suu:=u^2*S0;
```

$$Suu := 49.42437300 \quad \textbf{(37)}$$

Early Exercise:

```
> Suu-K+3;
```

$$7.42437300 \quad \textbf{(38)}$$

Problem 4. Check for convexity:

```
> P[1]:=5.0;P[2]:=12.0;P[3]:=15.0;K[1]:=30;K[2]:=40;K[3]:=60;
```

$$P_1 := 5.0$$
$$P_2 := 12.0$$

$$P_3 := 15.0$$

$$K_1 := 30$$

$$K_2 := 40$$

$$K_3 := 60 \quad \textbf{(39)}$$

```
> (P[2]-P[1])/(K[2]-K[1]);(P[3]-P[2])/(K[3]-K[2]);
```

$$0.7000000000$$

$$0.1500000000 \quad \textbf{(40)}$$

Since .7>.15, convexity is violated

Arbitrage (Butterfly spread)

```
> lam:=(K[3]-K[2])/(K[3]-K[1]);1-\lam;
```

$$lam := \frac{2}{3}$$

$$\frac{1}{3} \quad \textbf{(41)}$$

We buy 2/3 of the 30 strike and 1/3 of the 60 strike and sell 1 of the 40 strike. Or better, we buy 2 of the 30 strike and 1 of the 60 strike and sell 3 of the 40 strike. Minimal profit:

```
> -2*P[1]-P[3]+3*P[2];
```

$$11.0 \quad \textbf{(42)}$$

Problem 5

```
> C[1]:=20.0;C[2]:=5;K[1]:=30;K[2]:=40;
```

$$C_1 := 20.0$$

$$C_2 := 5$$

$$K_1 := 30$$

$$K_2 := 40 \quad \textbf{(43)}$$

```
> (C[1]-C[2]);(K[2]-K[1]);
```

$$15.0$$

$$10 \quad \textbf{(44)}$$

The first should be less than the second, so arbitrage exist. We by the cheapest, (40 strike), and sell the most expensive, 30 strike.. If S>40, we get S -40+30-S=-10. Our minimal profit is

```
> (C[1]-C[2])-10;
```

$$5.0 \quad \textbf{(45)}$$

Problem6, (a)

```
> Parity:=(r,h,del,S,K)->exp(-del*h)*S-exp(-r*h)*K;
```

$$Parity := (r, h, del, S, K) \rightarrow e^{-h\,del} S - e^{-h\,r} K \quad \textbf{(46)}$$

```
> S:=1.6;r:=.04;del:=.05;K:=1.5;h:=1;
```

$$S := 1.6$$

$$r := 0.04$$

$$del := 0.05$$

$$K := 1.5$$

$$h := 1 \quad \textbf{(47)}$$

Comput C-P

```
> CMP:=Parity(r,h,del,S,K);
```

$$CMP := 0.080782920 \quad \textbf{(48)}$$

Since C=.13,

```
> PD:=.13-CMP;
```

$$PD := 0.049217080 \quad \textbf{(49)}$$

Part (b) From formula (9.7), p. 292, xoKCE(1/xo,1/K,T)=PD(1/xo,1/K,T). Hence

```
> CD:=(1/S*K)*PD;
```

$$CD := 0.04614101250 \quad \textbf{(50)}$$

Problem 7

```
> S:=1.6;r:=.04;del:=.05;K:=1.5;sig:=.12;h:=1/12;
```

$$S := 1.6$$
$$r := 0.04$$
$$del := 0.05$$
$$K := 1.5$$
$$sig := 0.12$$
$$h := \frac{1}{12} \quad \textbf{(51)}$$

```
> ud(r,h,del,sig):
> u;d;
```

$$1.034385657$$
$$0.9651474904 \quad \textbf{(52)}$$

```
> p:=pq(r,h,del,u,d);q:=1-p;
```

$$p := 0.4913406156$$
$$q := 0.5086593844 \quad \textbf{(53)}$$

Problem 8. According to formula (9.2), p. 283, it is just the PV of the dividends i.e.

```
> PV:=exp(-.055*(1/4))*.90+exp(-.055*(2/4))*1.2+exp(-.055*(3/4))*
  1.45;
```

$$PV := 3.446563653 \quad \textbf{(54)}$$

```
>
```