STAT 473, Test 2, Fall 2008

All answers must be reported to at least 4 decimal accuracy.

(1) Assume the following data on a 6-month put option: $K = \$90.00$, $S = \$100$, $r = 5.0\%$, $u = 1.1$, $d = .8$
  (a) Draw the binomial tree, using 3-month intervals as the time period, filling in only the values of S.
  Ans: Row 2, 110.0, 80.0. Row 3, 121.00,88.00,64.00
  (b) What is the price of the option? Ans: 2.958889281
  (c) In our hedge at time $t = 0$, what are ΔS (the \$ value of the stock in our hedge) and B (the amount we borrow in our hedge)? Ans: $C = 2.958889281$, $B = 30.64699419$, $\Delta S = -27.68810491$, $\Delta = -0.2768810491$.
  (d) Answer the questions asked in part (c) for the node S_u.
  $C = 0.5755705339$, $B = 7.242237201$, $\Delta S = -6.666666667$, $\Delta = -0.06060606061$
  (e) Answer the questions asked in part (c) for the nodes S_d.
  $C = 8.882002008$, $B = 88.88200201$, $\Delta S = -80.00000000$, $\Delta = -1.000000000$

(2) We sell a European call with expiration in 1 year on a stock with spot price 50 and strike 45. Assume that the interest free interest rate is 5%, the continuously compounded dividend rate is 1% and the volatility is 10%. We price this option using a 4 step tree ($n = 4$).
  (a) Find p^* and q^*. $p = 0.4875026002$, $q = 0.5124973998$.
  (b) Find u and d. Ans: $u = 1.061836547$, $d = 0.9607894392$.
  (c) Given that the last row of the tree is 63.56245760, 57.51369005, 52.04053880, 47.08822670, 42.60718946, compute the price of the call. *Hint:* The binomial coefficients are 1,4,6,4,1. Ans: 6.854189495
  (d) Compute the value of the option at the node $S_u = uS$. *Hint* This is easily computed from the four largest entries in the last row of the tree. Ans: 9.351379338
  (e) Use the tree to compute the value of the corresponding European put. **Do not use Put-Call parity.** Ans: .1570219094
  (f) The second to last row of the tree is 59.86086825, 54.16435345, 49.00993369, 44.34602184. Is exercise of

the American option optimal at the 59.86086825 node? Explain. value at node 15.27040195, $59.86086825 - K = 14.86086825$. Ans: Do not exercise.

(g) In a 20 step tree for this option, what is the third largest entry in the last row? Ans = 74.42514660.

(3) Assume the same data as in Problem 2 **except** that instead of the dividend being compounded continuously, the stock pays a dividend of 3 in 6 months. We use the prepaid forward technique with a 4 step tree ($n = 4$) to price this option.

(a) What is the first ($t = 0$) entry in our tree? Ans: 47.07407026

(b) What is the volatility of the prepaid forward? Ans: .1062155869

(c) What is the value of early exercise of this call at the node S_{uu}? Ans: 7.42437300

(4) We observe the following prices P on three puts having strikes K. Assume that the puts all have the same expiration.

P	5	12	15
K	30	40	60

(a) Show that there is an arbitrage opportunity. Ans: Convexity is violated

(b) Describe the arbitrage technique. Ans: We buy 2/3 of the 30 strike and 1/3 of the 60 strike and sell 1 of the 40 strike. Or better, we buy 2 of the 30 strike and 1 of the 60 strike and sell 3 of the 40 strike.

(c) What is the smallest profit you would realize using this technique? Ans: The latter technique yields at least 11.

(5) We observe the following prices C on two calls having strikes K. Assume that the calls have the same expiration.

C	20	5
K	30	40

(a) Show that there is an arbitrage opportunity.

(b) Describe the arbitrage technique. Ans: Buy the 40 strike and sell the 30 strike

(c) What is the smallest profit you would realize using this technique? Ans: $15 - (40 - 30) = 5$.

(6) Suppose that the \$-denominated interest rate is 4%, the €-denominated interest rate is 5% and the spot exchange rate is 1.6\$/€. Suppose that the non-arbitrage price to buy a

\$ denominated call on 1€with one year to expiration and a strike price of 1.5\$ is .13\$.

(a) What is the non-arbitrage price of the corresponding put? Ans: 0.049217080

(b) What is the non-arbitrage price of a €-denominated call on 1\$ with the same expiration and a strike (2/3)€? Ans: 0.04614101250

(7) Let the assumptions be as in Problem 6. Assume that the volatility in the price of the Euro is 12% per year. In pricing the call option described in Problem 6 using a binomial tree with $n = 12$, what is p?, u?, d? Ans: $u = 1.034385657$, $d = 0.9651474904$, $p = 0.4913406156$, $q = 0.5086593844$

(8) A company is forecasted to pay dividends of \$0.90, \$1.20, and \$1.45 in 3, 6, and 9 months, respectively. Given interest rates of 5.5%, how much dollar impact will dividends have on the difference between the price of a put and a call on the companies' stock? (Assume a 9-month option and the same strike.)

Ans: 3.446563653