

Midterm #2

1. a) \$10,000 whole life (30), PTD w/ annual premiums, $i = 6\%$, ILT mortality

$$\text{Single payment premium} = 10,000 \bar{A}_{30} = 10,000 \frac{i}{\delta} A_{30} = 10,000 \frac{.06}{\ln(1.06)} \left(\frac{102.48}{1000} \right) = 1,055.25 \text{ (UDD)}$$

$$\text{b) Annual premium} = \frac{10,000 \bar{A}_{30}}{\ddot{a}_{30}} = \frac{1,055.25}{15.8561} = 66.55 \text{ (Again, UDD)}$$

$$\text{c) } L = 10,000 v^T - 66.55 \frac{1 - v^{K+1}}{d} = 10,000 (1.06)^{-T} - 66.55 \frac{1 - (1.06)^{-(K+1)}}{.06/1.06}$$

where T is the time until death and $K = \lfloor T \rfloor$

2. a) \$10,000, 20-yr term PTD to (30), $i = 6\%$, ILT mortality

$$\begin{aligned} \text{Single payment premium} &= 10,000 \bar{A}_{30:\overline{20}|}^1 = 10,000 (\bar{A}_{30} - {}_{20}E_{30} \bar{A}_{50}) = 10,000 \frac{i}{\delta} (A_{30} - {}_{20}E_{30} A_{50}) \\ &= 10,000 \frac{.06}{\ln(1.06)} \left(\frac{102.48}{1000} - \frac{293.74}{1000} \cdot \frac{249.05}{1000} \right) = 301.95 \text{ (UDD)} \end{aligned}$$

$$\text{b) } \ddot{a}_{30:\overline{20}|} = \ddot{a}_{30} - {}_{20}E_{30} \ddot{a}_{50} = 15.8561 - \frac{293.74}{1000} \cdot 13.2468 = 11.96$$

$$\text{Annual premium} = \frac{10,000 \bar{A}_{30:\overline{20}|}^1}{\ddot{a}_{30:\overline{20}|}} = \frac{301.95}{11.96} = 25.25 \text{ (UDD)}$$

$$\text{c) } \ddot{a}_{30}^{(12)} = \alpha(12) \ddot{a}_{30} - \beta(12) = 1.00028 (15.8561) - .46812 = 15.39$$

$$\ddot{a}_{50}^{(12)} = \alpha(12) \ddot{a}_{50} - \beta(12) = 1.00028 (13.2468) - .46812 = 12.80$$

$$\ddot{a}_{30:\overline{20}|}^{(12)} = \ddot{a}_{30}^{(12)} - {}_{20}E_{30} \ddot{a}_{50}^{(12)} = 15.39 - \frac{293.74}{1000} (12.80) = 11.63$$

$$\text{Annualized monthly premium} = \frac{10,000 \bar{A}_{30:\overline{20}|}^1}{\ddot{a}_{30:\overline{20}|}^{(12)}} = \frac{301.95}{11.63} = 25.96 \Rightarrow \text{monthly premium} = 2.16 \text{ (UDD)}$$

3. a) \$1,000 whole life PTD to (70), $\delta = .05$, $l_x = 100 - x$, $0 \leq x \leq 100$ (Note: De Moivre = UDD throughout)

$$\text{Single payment premium} = 1000 \bar{A}_{70} = 1000 \int_0^{70} e^{-\delta t} \frac{1}{70} dt = 1000 \left(\frac{1}{70} \right) \left(\frac{1}{.05} \right) (1 - e^{-.05(70)}) = 277.09$$

$$\text{b) } 277.09 = P + 2e^{-.05(10)} \left(\frac{60}{70} \right) + P e^{-.05(70)} \left(\frac{50}{70} \right) \Rightarrow P = 218.60$$

$$\begin{aligned} \text{c) } {}_5V &= 1000 \bar{A}_{35} - 2e^{-.05(5)} \left(\frac{60}{65} \right) - P e^{-.05(15)} \left(\frac{50}{65} \right) \\ &= 1000 \left(\frac{1}{65} \right) \left(\frac{1}{.05} \right) (1 - e^{-.05(65)}) - 2e^{-.05(5)} \left(\frac{60}{65} \right) - 218.60 e^{-.05(15)} \left(\frac{50}{65} \right) = 214.89 \end{aligned}$$

4. a) \$300 whole life PTD to (30), $\delta = .05$, $l_x = 100 - x$, paid w/ a continuous annuity (De Moivre = UDD)

$$L = 300 v^T - \bar{P}(\bar{A}_{30}) \frac{1 - v^T}{\delta}, \quad \bar{a}_{30} = \frac{1 - \bar{A}_{30}}{\delta} = \frac{1 - .27709}{.05} = 14.46 \text{ (using values from 3a)}$$

$$\bar{P}(\bar{A}_{30}) = \frac{300 \bar{A}_{30}}{\bar{a}_{30}} = \frac{277.09}{14.46} = 19.16$$

$$\text{Thus, } L = 300 e^{-.05T} - 19.16 \frac{1 - e^{-.05T}}{.05}$$

b) - MIXED -

$$c) \text{Var}(L) = \text{Var}\left(1000 v^T - 19.16 \frac{1-v^T}{.05}\right) = \text{Var}\left(\left(1000 + \frac{19.16}{.05}\right) v^T\right) = \left(1000 + \frac{19.16}{.05}\right)^2 \text{Var}(v^T)$$

$${}^2\bar{A}_{30} = \int_0^{30} e^{-.1t} \frac{1}{30} dt = \left(\frac{1}{30}\right)\left(\frac{1}{.1}\right)(1 - e^{-.1(30)}) = .1427, \quad \bar{A}_{30} = .27709 \quad (\text{from 3a})$$

$$\Rightarrow \text{Var}(L) = \left(1000 + \frac{19.16}{.05}\right)^2 ({}^2\bar{A}_{30} - (\bar{A}_{30})^2) = \left(1000 + \frac{19.16}{.05}\right)^2 (.1427 - (.27709)^2) = 126,194.92$$

$$d) {}_{10}L = 1000 v^{T-10} - 19.16 \frac{1-v^{T-10}}{.05} = 1000 e^{-.05(T-10)} - 19.16 \frac{1-e^{-.05(T-10)}}{.05} \quad \text{where } T \text{ is the future lifetime of } (30)$$

$$e) \text{Var}({}_{10}L | T(30) \geq 10) = \left(1000 + \frac{19.16}{.05}\right)^2 ({}^2\bar{A}_{40} - (\bar{A}_{40})^2)$$

$$\bar{A}_{40} = \int_0^{60} e^{-.05t} \frac{1}{60} dt = \frac{1}{60} \left(\frac{1}{.05}\right)(1 - e^{-.05(60)}) = .3167, \quad {}^2\bar{A}_{40} = \int_0^{60} e^{-.1t} \frac{1}{60} dt = \frac{1}{60} \left(\frac{1}{.1}\right)(1 - e^{-.1(60)}) = .1663$$

$$\Rightarrow \text{Var}({}_{10}L | T(30) \geq 10) = \left(1000 + \frac{19.16}{.05}\right)^2 (.1663 - (.3167)^2) = 126,158.25$$

$$5, \quad (x = 100, \quad l_{x+1} = 90, \quad l_{x+2} = 60, \quad i = .04)$$

$${}_0V + 50 = v q_x(1000) + v p_x({}_1V) \Rightarrow 50 = (1.04)^{-1}(.1)(1000) + (1.04)^{-1}(.9)({}_1V) \Rightarrow {}_1V = -53.33$$

$${}_1V + 100 = v q_{x+1}(1000) + v p_{x+1}({}_2V) \Rightarrow -53.33 + 100 = (1.04)^{-1}\left(\frac{1}{3}\right)(1000) + (1.04)^{-1}\left(\frac{2}{3}\right)({}_2V) \Rightarrow {}_2V = -427.2$$

$$6, \quad \delta = .05, \quad \bar{A}_{50} = .25, \quad {}_{10}\bar{V}(\bar{A}_{40}) = .11765, \quad B \bar{A}_{40} = 25.93 \ddot{a}_{40}, \quad \text{Find } B.$$

$${}_{10}\bar{V}(\bar{A}_{40}) = \frac{\bar{A}_{50} - \bar{A}_{40}}{1 - \bar{A}_{40}} \Rightarrow .11765 = \frac{.25 - \bar{A}_{40}}{1 - \bar{A}_{40}} \Rightarrow .11765 - .11765 \bar{A}_{40} = .25 - \bar{A}_{40} \Rightarrow \bar{A}_{40} = .15$$

$$A_{40} = \frac{\delta}{i} \bar{A}_{40} = \frac{.05}{e^{.05} - 1} (.15) = .146, \quad \ddot{a}_{40} = \frac{1 - A_{40}}{d} = \frac{1 - .146}{1 - e^{-.05}} = 17.505$$

$$\Rightarrow B(.15) = 25.93(17.505) \Rightarrow B = 3026.06 \quad (\text{UDD})$$

~~assume UDD~~

$$\mu_x = .05, \mu_y = .04, \delta = .06$$

$$\bar{A}_{x:\overline{10}|} = \int_0^{10} e^{-.06t} e^{-.05t} (.05) dt = \frac{.05}{.11} (1 - e^{-.11(10)})$$

$$\bar{A}_{y:\overline{10}|} = \int_0^{10} e^{-.06t} e^{-.04t} (.04) dt = \frac{.04}{.1} (1 - e^{-.1(10)})$$

$$\bar{A}_{xy:\overline{10}|} = \int_0^{10} e^{-.06t} e^{-.09t} (.09) dt = \frac{.09}{.15} (1 - e^{-.15(10)})$$

$$+P_{\overline{xy}} = +P_x + +P_y - +P_{xy} = e^{-.05t} + e^{-.04t} - e^{-.09t}$$

$$-(+P_{\overline{xy}})' = .05 e^{-.05t} + .04 e^{-.04t} - .09 e^{-.09t}$$

$$\bar{A}_{\overline{xy}:\overline{10}|} = \int_0^{10} e^{-.06t} (.05 e^{-.05t} + .04 e^{-.04t} - .09 e^{-.09t}) dt$$

$$= \left(-\frac{.05}{.11} e^{-.11t} - \frac{.04}{.10} e^{-.1t} + \frac{.09}{.15} e^{-.15t} \right) \Big|_0^{10}$$

$$= \frac{.05}{.11} (1 - e^{-.11(10)}) + \frac{.04}{.10} (1 - e^{-.1(10)}) - \frac{.09}{.15} (1 - e^{-.15(10)})$$

$$\bar{A}_{xy} + \bar{A}_{\overline{xy}} = \bar{A}_x + \bar{A}_y$$

$$\bar{A}'_{\overline{xy}:\overline{10}|} = ? \quad \bar{A}'_{xy:\overline{10}|} + \bar{A}'_{\overline{xy}:\overline{10}|} = \bar{A}'_{x:\overline{10}|} + \bar{A}'_{y:\overline{10}|}$$

$$- \bar{A}'_{x:\overline{10}|} = \bar{A}_x - v^{10} {}_{10}p_x \bar{A}_{x+10}$$

$$- \bar{A}'_{y:\overline{10}|} = \bar{A}_y - v^{10} {}_{10}p_y \bar{A}_{y+10}$$

$$- \bar{A}'_{xy:\overline{10}|} = \bar{A}_{xy} - v^{10} {}_{10}p_x {}_{10}p_y \bar{A}_{x+10:y+10}$$

$$\Rightarrow \bar{A}'_{\overline{xy}:\overline{10}|} = \bar{A}'_{x:\overline{10}|} + \bar{A}'_{y:\overline{10}|} - \bar{A}'_{xy:\overline{10}|}$$

$$= \bar{A}_x - v^{10} {}_{10}p_x \bar{A}_{x+10} + \bar{A}_y - v^{10} {}_{10}p_y \bar{A}_{y+10} - \bar{A}_{xy} + v^{10} {}_{10}p_x {}_{10}p_y \bar{A}_{x+10:y+10}$$

$$= (\bar{A}_x + \bar{A}_y - \bar{A}_{xy}) - v^{10} ({}_{10}p_x \bar{A}_{x+10} + {}_{10}p_y \bar{A}_{y+10} - {}_{10}p_x {}_{10}p_y \bar{A}_{x+10:y+10})$$

$$\neq \bar{A}_{\overline{xy}} - v^{10} ({}_{10}p_x + {}_{10}p_y - {}_{10}p_x {}_{10}p_y) \bar{A}_{x+10:y+10}$$

$$({}_{10}p_x \cdot {}_{10}p_y - {}_{10}p_x {}_{10}p_y) (\bar{A}_{x+10} + \bar{A}_{y+10} - \bar{A}_{x+10:y+10})$$

$$= {}_{10}p_x \bar{A}_{x+10} + {}_{10}p_y \bar{A}_{y+10} - {}_{10}p_x \bar{A}_{x+10:y+10} - {}_{10}p_y \bar{A}_{x+10:y+10} + {}_{10}p_x {}_{10}p_y \bar{A}_{x+10:y+10}$$