Eddie Price **Quiz 15 Solutions** **Spring 2017**

High score: 10; Non-0 Low score: 2; Average score: 7.44 (including 0's)

Problem 1 (5 Points). Evaluate the double integral

$$\int_{-1}^{0}\int_{y}^{2} (x+y)\ dx\, dy$$

Solution. We do the inside integral first, which is an integral of x so we treat y as a constant

$$\int_{-1}^{0} \left(\frac{1}{2}x^2 + xy\right)\Big|_{x=y}^{x=2}\ dy$$

$$= \int_{-1}^{0} \left(2 + 2y - \frac{1}{2}y^2 - y^2\right)\ dy$$

$$= \int_{-1}^{0} \left(2 + 2y - \frac{3}{2}y^2\right)\ dy$$

$$= \left(2y + y^2 - \frac{1}{2}y^3\right)\Big|_{-1}^{0}$$

$$= 0 + 0 - 0 - \left(-2 + 1 + \frac{1}{2}\right) = -\left(-\frac{1}{2}\right) = \boxed{\frac{1}{2}}$$

Problem 2 (5 Points). Evaluate the double integral

$$\int_{0}^{\sqrt{\pi/4}}\int_{0}^{\sqrt{x}} 4y\cos\left(x^2\right)\ dy\, dx$$

Solution. We do the inside integral first, which is an integral of y so we treat x as a constant

$$= \int_{0}^{\sqrt{\pi/4}} 2y^2\cos\left(x^2\right)\big|_{0}^{\sqrt{x}}\ dx$$

$$= \int_{0}^{\sqrt{\pi/4}} 2x\cos\left(x^2\right)\ dx$$

We do this by substitution. Let $u = x^2$. Then $du = 2x\, dx$. Also, $u(0) = 0^2 = 0$ and $u\left(\sqrt{\frac{\pi}{4}}\right) = \frac{\pi}{4}$, so we get the integral

$$= \int_{0}^{\pi/4} \cos(u)\ du$$

$$= \sin(u)|_{0}^{\pi/4} = \sin\left(\frac{\pi}{4}\right) - \sin(0) = \boxed{\frac{\sqrt{2}}{2}}$$

Common Mistakes

In problem 1, people forgot how to integrate y with respect to x. y is treated like a constant in an integral with respect to x, so the integral is xy (just like how the integral of 5 is $5x$). Many people put either y (forgetting to integrate) or 0 (differentiating instead of integrating).

In problem 1, many people plugged the bounds of the integral into the integral in the wrong order. Recall that $\int_a^b f'(x)\ dx = f(b) - f(a)$, so you plug in the upper bound first.

In problem 1, many people forgot that negative numbers cube to negative numbers. Also, some people forgot to distribute their negative signs when necessary.

In problem 2, many people plugged the bounds for the y-integral into both x and y. They should only be plugged into y since we are treating x like a constant in the y integral.

In problem 2, many people tried to integrate $2x\cos(x^2)$ by integrating the factors separately and obtaining $x^2\sin(x^2)$. This is wrong. This is not how integration works. This is the reason we need things like u-substitution and integration by parts.

In problem 2, many people tried to integrate $2x\cos(x^2)$ using integration by parts. This only makes the integral more complicated. Use u-substitution.

In problem 2, many people got the sign wrong for the integral of $\cos(u)$ saying it was $-\sin(u)$. Remember that the derivative of sine is cosine, so the integral of cosine is just sine.

In problem 2, many people rounded their answer to several decimal places. Please keep answers exact unless otherwise specified.