Math 266 Summer 2016 Quiz 5

1) Find the general solution of the differential equation $4y'' + 17y' + 4y = 0$.

$4r^2 + 17r + 4 = 0$

$r = \dfrac{-17 \pm \sqrt{289 - 64}}{8}$

$r = \dfrac{-17 \pm \sqrt{225}}{8}$

$= \dfrac{-17 \pm 15}{8}$

roots are $-\frac{1}{4}$ and -4

$y(t) = c_1 e^{-t/4} + c_2 e^{-4t}$

2) Find a second solution using reduction of order.

$t^2y'' + 2ty' - 2y = 0; \quad y_1(t) = t, t > 0$

$y = vy_1 = tv$

$y' = tv' + v$

$y'' = tv'' + v' + v' = tv'' + 2v'$

$t^2(tv'' + 2v') + 2t(tv' + v) - 2(tv) = 0$

$t^3v'' + 2t^2v' + 2t^2v' + 2tv - 2tv = 0$

$t^3v'' + 4t^2v' = 0$

Let $u = v'$. Then $u' = v''$

$t^3u' + 4t^2u = 0$

$u' + \frac{4}{t}u = 0$

$\mu(t) = \exp\left(\int \frac{4}{t}\,dt\right) = (e^{\ln t})^4 = t^4$

$\frac{d}{dt}[t^4u] = 0$

$t^4u = k_2$

$u = k_2t^{-4}$

$v' = k_2t^{-4}$

$v = \frac{k_2}{-3}t^{-3} + c_1$

$= c_2t^{-3} + c_1$

$y = vt = (c_2t^{-3} + c_1)t$

$= c_2t^{-2} + c_1t$

So $y_2 = t^{-2}$ or $\frac{1}{t^2}$

high: 20
low: 11
mean: 15.89