Math 266, Practice Midterm 1

Instructions for the real exam: This is a 1-hour exam. No calculators or notes are allowed. Please show your work (except on multiple choice questions).

1. Which of the following differential equations is separable?

   (a) $\frac{dy}{dx} = \frac{x^2+xy+y^2}{2y^2}$

   (b) $-2\ln(x) + 3yy' = 0$

   (c) $(y + 3\cos(y))\,dx + (x - 3x\sin(y))\,dy = 0$

   (d) $y' + 3\ln(x)y = 2x^2 + e^x$

2. Which of the following functions is the integrating factor for the equation $ty' = 2y + t^3$?

   (a) e^{2t}

   (b) e^{-2t}

   (c) $2/t$

   (d) $2t$

3. A large tank is full of a corrosive fluid, which is leaking out of a small circular hole at a rate proportional to the fourth power of the hole's radius. As the fluid leaks, it corrodes the edges of the hole and enlarges it. The area of the hole increases at a rate proportional to the amount of fluid that has leaked out. Write (don't solve!) a differential equation that describes this situation.

4. Solve the following initial value problem, and give the maximum interval on which the solution is valid.

$$ty' + 4y = t^{-2}e^t, \quad y(1) = 2.$$

5. Solve the following initial value problem, and give the maximum interval on which the solution is valid.

$$y' = \frac{4x}{1+2y}, y(1) = -1.$$

6. Find all linear solutions (functions of the form $y = mx + b$) to the differential equation

$$\frac{dy}{dx} = \frac{x^2 - 2y^2}{2xy}.$$

7. An Indonesian island can support a population of up to 600 cassowaries (see below). The birds only reproduce enough to grow the population if there are more than 100 of them – if there are any fewer, the population begins to die out.

(a) Write a differential equation that describes the population of cassowaries on the island. Define any symbols you introduce.

(b) Sketch a direction field for your equation.

8. Consider the differential equation

$$xy^2 + Ax^2y + (x^3 + yx^2)\frac{dy}{dx} = 0.$$

Find the value of A that makes the equation exact. Solve the equation for this value of A.

9. Use the substitution $v = y^3$ to find the general solution to the differential equation

$$y' = 5y + e^{-2x}y^{-2}.$$