Math 266, Practice Midterm 2

This is a 1-hour exam. No calculators or notes are allowed. Please show your work (except on multiple choice questions). Each multiple choice question has a single correct answer. If you finish early, you may bring your exam up to the front and leave the room.

Name: ____

Section: MWF 3:30-4:30 MWF 4:30 - 5:30

Useful things to remember: 1 N = 1 Newton = $1\,\mathrm{kg}\cdot\mathrm{m/s}^2$.

$$\cos(\alpha+\beta) = \cos(\alpha)\cos(\beta) - \sin(\alpha)\sin(\beta).$$

$$\sin(\alpha+\beta) = \sin(\alpha)\cos(\beta) + \cos(\alpha)\sin(\beta).$$

$$e^{it} = \cos(t) + i\sin(t).$$

It's all right to leave answers that would require complicated arithmetic in unsimplified form.

1. Consider a differential equation of the form

$$y'' + p(t)y' + q(t)y = g(t). \tag{1}$$

Suppose that $\{y_1, y_2\}$ are an fundamental set of complex-valued solutions to the associated homogeneous equation

$$y'' + p(t)y' + q(t)y = 0, \tag{2}$$

with $y_2 = \overline{y_1}$. Also suppose that $y = t^2$ is a solution to (1). What is the general real-valued solution to (1)?

(a) $y = t^2 + C_1 y_1 + C_2 y_2$

(b) $y = t^2 + C_1 \mathrm{Re}(y_1) + C_2 \mathrm{Re}(y_2)$

(c) $y = t^2 + C_1 \mathrm{Re}(y_1) + C_2 \mathrm{Im}(y_1)$

(d) $y = t^2 + C_1 \mathrm{Re}(y_1) + C_2 \mathrm{Im}(y_1) + C_3 \mathrm{Re}(y_2) + C_4 \mathrm{Im}(y_2)$

2. Consider the first-order initial value problem

$$y' = t - y^2, \quad y(0) = y_0.$$

What is the approximate value for $y(2)$ computed by Euler's method with step size 1?

(a) $y(2) \approx y_0 - y_0^2 + 1 - (y_0 - y_0^2)^2$

(b) $y(2) \approx 2 - y_0^2$

(c) $y(2) \approx 1 - (y_0 - y_0^2)^2$

(d) $y(2) \approx y_0 + 2 - y_0^2 - (y_0 + 1 - y_0^2)^2$

3. An RLC circuit has a resistor with a variable resistance, R. The current I is described by the formula
$$4I'' + RI' + I/4 = 0.$$
For what values of R will the current decrease over time and oscillate as it does so?
   (a) $0 < R < 1$
   (b) $0 < R < 2$
   (c) $0 \leq R \leq 2$
   (d) $0 < R$
   (e) $R \geq 2$

4. Consider a differential equation of the form
$$y'' + \alpha y' + 4y = e^{2t}.$$
For what value(s) of α will the equation have a solution of the form $y = Ate^{2t}$?
   (a) $\alpha = -4$
   (b) $\alpha = 4$
   (c) $\alpha = \pm 4$
   (d) There is no such value of α.

5. The tides at Cardiff oscillate according to the formula

$$y(t) = (5\,\mathrm{in})\cos(t/(12\,\mathrm{hr})) + (1\,\mathrm{ft})\cos(t/(12\,\mathrm{hr}).$$

(a) What are the amplitude and period of the motion?

(b) What is the first time after $t = 0$ at which the tide is at its maximum?

6. Find the general solution to the equation

$$t^2 y'' + t(t-3)y' - (t-3)y = 0, \quad t > 0.$$

(Hint: one solution is $y = t$.)

7. Find any solution to the equation

$$y'' + y = 1 + \tan(x), \quad -\pi/2 < x < \pi/2.$$

8. A 1 kg mass stretches a spring 0.4 m. The mass-spring system starts at equilibrium and is acted on by a variable force $F_{\text{ext}}(t) = \cos(5t)$. Write the equation describing the displacement of the mass as a function of time, and describe in words what happens to the spring. You may assume that the spring is undamped and $g = 10\,\text{m/s}^2$.

(Scratch paper)

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