# Math 303, Homework 1

## August 23, 2019

*Remember to show your work and name your collaborators. This page has a front and a back.*

0. Think about the following concepts from linear algebra and differential equations:

   - determinants;
   - linear independence and dependence;
   - eigenvectors and eigenvalues;
   - complex numbers;
   - how to solve a single first-order ODE with constant coefficients;
   - how to solve a single second-order ODE with constant coefficients.

   If you feel rusty on any of these concepts, find some problems about it (in our textbook, a different textbook, or online) and do them. *(You don't have to hand any of this in, but you do have to do it.)*

1. Find a solution to the initial value problem:

$$x_1' = 3x_1 - 4x_2, \qquad\qquad x_1(0) = 2,$$
$$x_2' = 4x_1 + 3x_2, \qquad\qquad x_2(0) = 2.$$

2. Find a system of the form $\mathbf{x}' = A\mathbf{x}$, where $A$ is a constant coefficient matrix, such that two solutions are

$$\mathbf{x}(t) = \begin{pmatrix} 3 \\ 2 \end{pmatrix} e^{4t} \quad \text{and} \quad \mathbf{x}(t) = \begin{pmatrix} 6 \\ 4 \end{pmatrix} e^{4t} + \begin{pmatrix} 1 \\ -1 \end{pmatrix} e^{-t}.$$

For each of the following scenarios, write a differential equation, or system of differential equations, describing the situation. Make sure to say explicitly what all your variables mean, any relevant initial values, and any important assumptions you make. Then, make an educated guess as to what will happen to the variables over time. You don't need to solve your equations.

3. A 10-kg mass hangs from a ceiling by a spring with spring constant 2 N/m. Another 10-kg mass hangs from the first mass by another spring with spring constant 1 N/m. Both springs are undamped.

4. A forest contains deer and wolves. If there were no wolves, the deer population would grow at a rate proportional to the current population. If there were no deer, the wolf population would decrease at a rate proportional to the current population. When the two animals live together, each wolf eats a constant amount of deer per year, and this increases the population of wolves at a rate proportional to both the deer population and the wolf population.

5. A house has a main floor, which is insulated from the cold outdoors, and an attic, which is not. Heat flows from the main floor to the attic, and from the attic to the outdoors, following Newton's law of cooling: the rate of temperature change of one space caused by its interaction with another space is proportional to the temperature difference. Initially, the temperature outside is 40° F, and the temperature in the attic and main floor is 50° F. There is also a heater on the main floor which increases its temperature by 20° F per hour.