Math 303, Homework 2

August 29, 2019

Remember to explain your reasoning and show your work!

0. Download the Java program `pplane` from `https://math.rice.edu/~dfield/dfpp.html`, and play around with it a little bit. Clicking on the graph will draw a solution to the system with those initial values. Try entering some of the systems from the homework – do you notice any patterns? Can you make any curves that look especially weird? *(You don't need to hand in anything for this one.)*

1. Find the general solution to the system

$$\mathbf{x}' = \begin{pmatrix} 14 & 9 \\ -25 & -16 \end{pmatrix}.$$

What happens to solutions to this system as $t \to \infty$?

2. (a) Find a 3×3 matrix A such that 0 is the only eigenvalue of A, and the space of eigenvectors of 0 has dimension 1. *(Hint: upper triangular matrices are your friend!)*

   (b) Find the general solution to $\mathbf{x}' = A\mathbf{x}$.

3. A 0.5-kg mass is attached to a spring with spring constant 2.5 N/m. The spring experiences friction, which acts as a force opposite and proportional to the velocity, with magnitude 2 N for every m/s of velocity. The spring is stretched 1 meter and then released.

   (a) Find a formula for the position of the mass as a function of time.

   (b) How much time does it take the mass to complete one oscillation (to pass the equilibrium point, bounce back, and return travelling in the same direction)?

(c) By what fraction has the amplitude of the motion decreased in this time?

(d) Do the answers to (b) and (c) depend on the initial position of the mass? Why or why not?

(e) By immersing the spring in one of a variety of rare, delicious syrups, it's possible to increase the damping constant while keeping the spring constant the same. Can you increase the damping constant so that the spring doesn't oscillate at all, but just returns to its starting point? What's the smallest value of the damping constant that will do this?