Math 303, Homework 3

Due September 12, 2019

Remember to explain your reasoning and show your work!
For problems 1 and 2:

(a) Produce a graph of the phase plane, and some representative solutions (you can do this in pplane and print it, or sketch it by hand);

(b) Say what you can about the eigenvalues of the matrix, from looking at the graph;

(c) Describe the critical behavior at 0: is it a node, a center or a spiral point? a sink or a source? if a node, is it proper or improper?

1.
$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 2 & 5 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}.$$

2.
$$\begin{aligned} x' &= 1.4x + 2.4y, \\ y' &= -1.8x - 2.6y. \end{aligned}$$

3. A complicated mechanical system has four moving parts. By analyzing the system, you've determined that the positions of the parts are related by the differential equations:

$$\begin{aligned} x_1' &= 2.03x_1 + 3.34x_2 - 0.28x_3 - 0.41x_4, \\ x_2' &= 0.49x_1 + 1.33x_2 + 0.14x_3 + 0.21x_4, \\ x_3' &= 7.13x_1 - 9.23x_2 + 5.32x_3 + 2.49x_4, \\ x_4' &= -2.92x_1 + 1.09x_2 - 1.12x_3 + 1.32x_4. \end{aligned}$$

What will happen to the parts when you put the system in motion? *(You probably want to use a computer for this. If you're not sure how, you should take a look at "Application 5.2" on page 295 of the book.)*