Math 303, Homework 3 solutions

For problems 1 and 2:

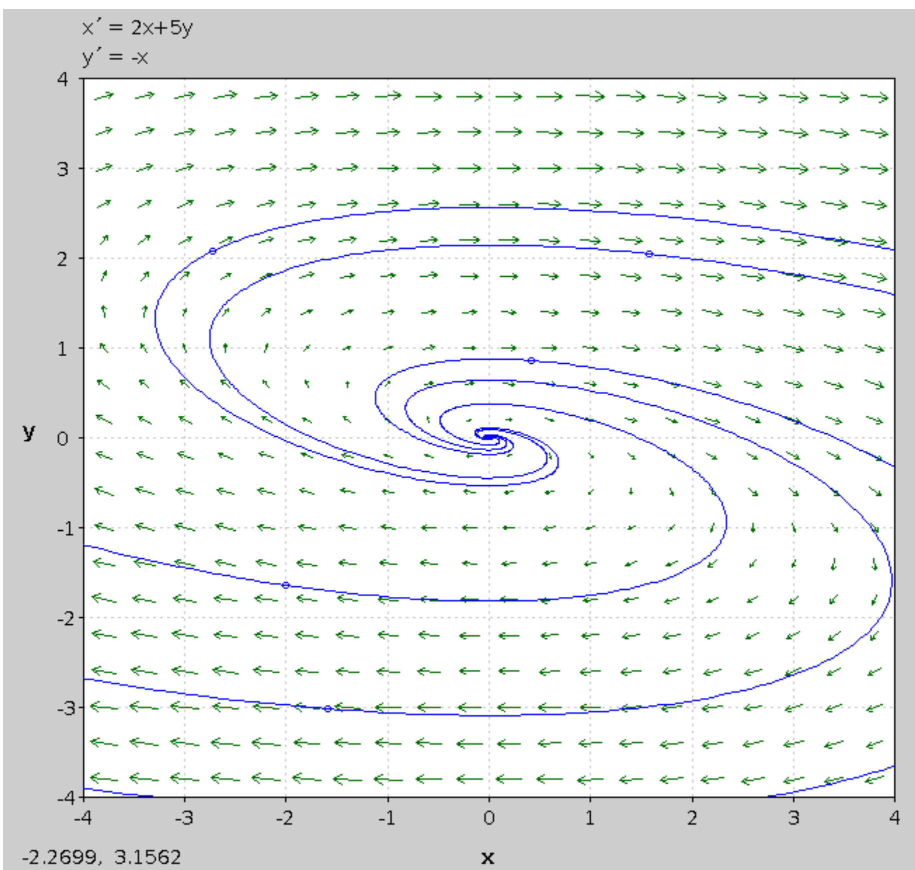
(a) *Produce a graph of the phase plane, and some representative solutions (you can do this in* pplane *and print it, or sketch it by hand);*

(b) Say what you can about the eigenvalues of the matrix, from looking at the graph;

(c) Describe the critical behavior at 0: is it a node, a center or a spiral point? a sink or a source? if a node, is it proper or improper?

1.

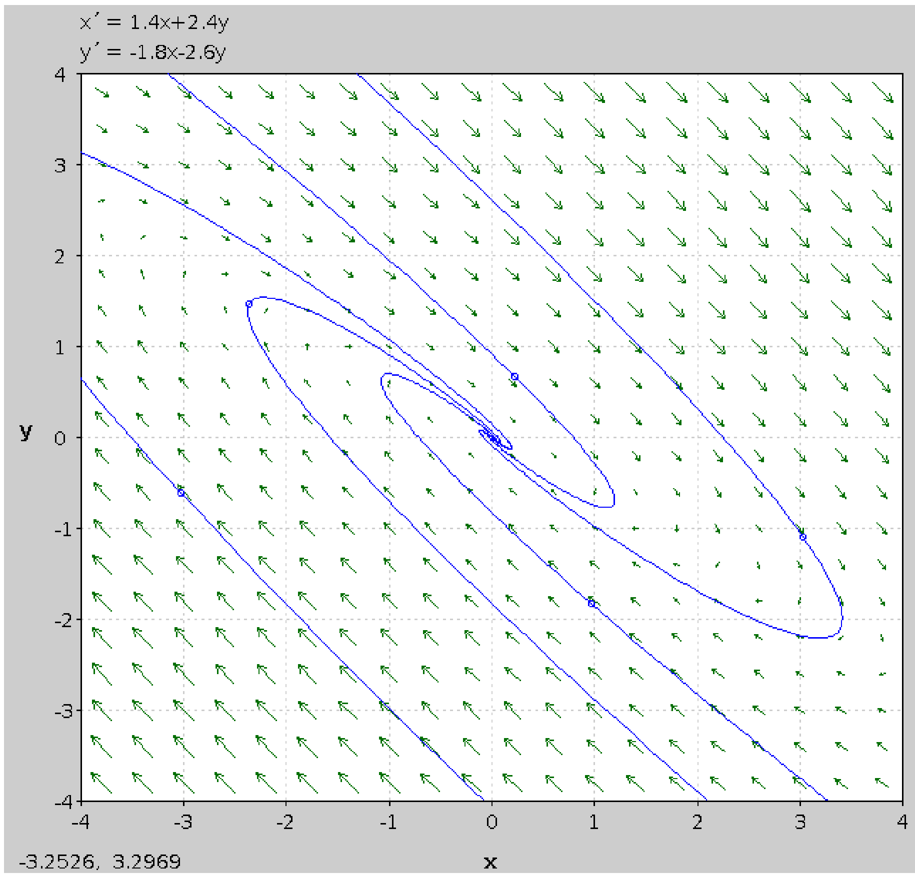
$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 2 & 5 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}.$$



The critical point at 0 is a **source** and a **spiral point**. Thus, the eigenvalues are **complex with positive real part**.

2.

$$x' = 1.4x + 2.4y,$$
$$y' = -1.8x - 2.6y.$$



The critical point at 0 is a **sink** and a **spiral point**. Thus, the eigenvalues are **complex with negative real part**.

3. *A complicated mechanical system has four moving parts. By analyzing the system, you've determined that the positions of the parts are related by the differential equations:*

$$x_1' = 2.03x_1 + 3.34x_2 - 0.28x_3 - 0.41x_4,$$
$$x_2' = 0.49x_1 + 1.33x_2 + 0.14x_3 + 0.21x_4,$$
$$x_3' = 7.13x_1 - 9.23x_2 + 5.32x_3 + 2.49x_4,$$
$$x_4' = -2.92x_1 + 1.09x_2 - 1.12x_3 + 1.32x_4.$$

What will happen to the parts when you put the system in motion? (You probably want to use a computer for this. If you're not sure how, you should take a look at "Application 5.2" on page 295 of the book.)

I entered into WolframAlpha:

$$((\mathsf{2.03, 3.34, -0.28, 0 - 0.41}), (\mathsf{0.49, 1.33, 0.14, 0.21}), \\ (\mathsf{7.13, -9.23, 5.32, 2.49}), (\mathsf{-2.92, 1.09, -1.12, 1.32}))$$

The eigenvalues it returned were

$$\lambda_1 \approx 3.97245,\ \lambda_2 \approx 3.01342,\ \lambda_3 \approx 2.00787, \lambda_4 \approx 1.00625.$$

These are all positive real eigenvalues, which means, under any nonzero initial conditions, that all four variables will go to $\pm\infty$ with increasing time. (How they do this, and whether each one goes to $+\infty$ or $-\infty$, depends on what the initial conditions are.) This is pretty bad behavior for most mechanical systems, and probably means either that the system will break or that this linear model for it will stop being accurate at some point.