Math 303, Homework 5

Due October 3, 2019

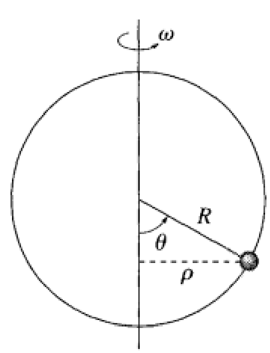
Note: You have learned enough to start handling some serious problems now, and these problems are pretty serious! If you're having trouble, you might want to see if there's anything relevant in the book, ask your classmates, post on the Blackboard message board, or see what others have posted there. For both problems, you need to explain your reasoning for all the conclusions you come to, and write anything that's not just a calculation out in complete sentences.

1. Consider a competing-species system of the form

$$x' = a_1 x - b_1 x^2 - c_1 xy,$$
$$y' = a_2 y - b_2 y^2 - c_2 xy,$$

where all the constants are positive.

Qualitatively speaking, there are a few different types of behavior such a system can have. For example, one possibility is that the species converge on a stable equilibrium at which both have nonzero population. Give a classification of the possible behaviors of this system, and say which values of the constants lead to which behavior.



2. A hula hoop of radius R is stood on its end and rotated around the vertical axis with constant angular frequency ω. Inside the hula hoop is a marble of mass

m, which can only move in the circle of the hula hoop. (See the above diagram, which I cribbed from Taylor's *Classical Mechanics*, p. 261, the same place I got the idea for this problem.) Writing θ for the marble's angle up from the vertical axis (so that $\theta = 0$ is the bottom of the hula hoop), the equation of motion for the marble is

$$\theta'' = \left(\omega^2 \cos(\theta) - \frac{g}{R}\right) \sin(\theta). \tag{1}$$

You don't have to derive this equation (though you're welcome to try, or look at how Taylor does it), but the idea is that, from the point of view of the hoop, the marble experiences both gravity and a centrifugal "force" coming from the rotation of the hoop that pushes it outwards from the axis of rotation.

(a) Find all the critical points of the system, determine the type and stability of each, and give a physical interpretation of your results. Your answer may depend on the values of the parameters.

(b) At each center of the system, find the period of oscillations of the linearization. This is approximately the period of small oscillations of the nonlinear system around that point.

(c) If ω is large compared to g/R, there is another kind of closed orbit that doesn't just circle around a single center. You should be able to find this orbit in `pplane`. What is the marble doing in these orbits?

(d) Use Matlab and `ode45` to find the period of these more complicated orbits, for some different choices of the parameters and initial values. Either come up with an approximate formula for these periods, or just describe how they depend on the parameters.