Math 303, Homework 6

Due October 10, 2019

1. (a) The Fourier series

$$\sin(t) - \frac{1}{2}\sin(2t) + \frac{1}{3}\sin(3t) - \frac{1}{4}\sin(4t) + \cdots$$

converges to a 2π-periodic function with a much simpler description. Figure out what function it is, and prove your claim.

   (b) By evaluating your result at an appropriate number, come up with a cool formula of the form
$$\pi/4 = (\text{something}).$$

2.

Come, investigate loneliness!
a solitary leaf
clings to the Kiri tree

–Basho

This leaf can be modelled as an undamped spring with mass 1 g and spring constant 0.5 g/s^2. Every two seconds, a dewdrop of mass 0.01 g lands on the leaf, and remains there for 1 second before sliding off. What (approximately) is the furthest that the leaf is displaced from equilibrium? *(Hint: what forces are acting on the leaf?)*