Math 303, Homework 8

Due October 24, 2019

1. This problem only requires trig, but I've alluded to it often enough in class that you might as well do it. Suppose that you have a linear combination of cosines and sines of the same frequency, of the form

$$A\cos(\omega t) + B\sin(\omega t).$$

Show that this can be written as a single, phase-shifted sine function, of the form

$$R\sin(\omega t + \delta).$$

(In particular, you have to find R and δ.)

2. Suppose that a spring-mass system with mass 1 kg, damping constant 1 kg/s, and spring constant 1 kg/s^2 is forced by a sinusoidal force $F(t) = \sin(\omega t)$. Here ω is left as a parameter, so your answers will depend on ω.

   (a) Find a formula for the steady-state motion of the mass.

   (b) Using the previous problem, find the amplitude and phase shift of the motion. (The phase shift is δ in the previous problem – this is the number of radians by which the mass lags behind the force.)

   (c) What value of ω (exactly) gives a steady-state motion of maximum amplitude?

3. Suppose that the system in the previous problem is forced by the π-periodic square wave, defined on the interval $[0, \pi)$ by

$$F(t) = \begin{cases} 1 & 0 \le t < \pi/2, \\ -1 & \pi/2 \le t < \pi. \end{cases}$$

Find a formula for the steady-state motion of the mass.