# Math 303, Practice midterm 2

**Instructions:** *The real exam is 1 hour long and has 6 questions. No calculators or notes will be permitted. For each question, the correct multiple-choice answer gets you full credit. For an incorrect answer, you will get partial credit based on the work you do towards the answer.*

1. The equation of motion of a damped pendulum is

$$\theta'' + \gamma\theta' + \frac{g}{L}\sin(\theta) = 0,$$

where $L$ is the pendulum's length, $\gamma$ is the damping constant, $g > 0$ is the acceleration due to gravity, and $\theta$ is the angle of the pendulum, measured so that $\theta = 0$ when the pendulum is at the bottom of its swing.

Suppose that the damping constant $\gamma$ is fixed, but we can adjust the pendulum's length $L$. Describe the critical behavior at $\theta = 0$.

(a) A nodal sink for all values of $L$

(b) A spiral sink for all values of $L$

(c) A center if $L$ is small, and a nodal sink if $L$ is large

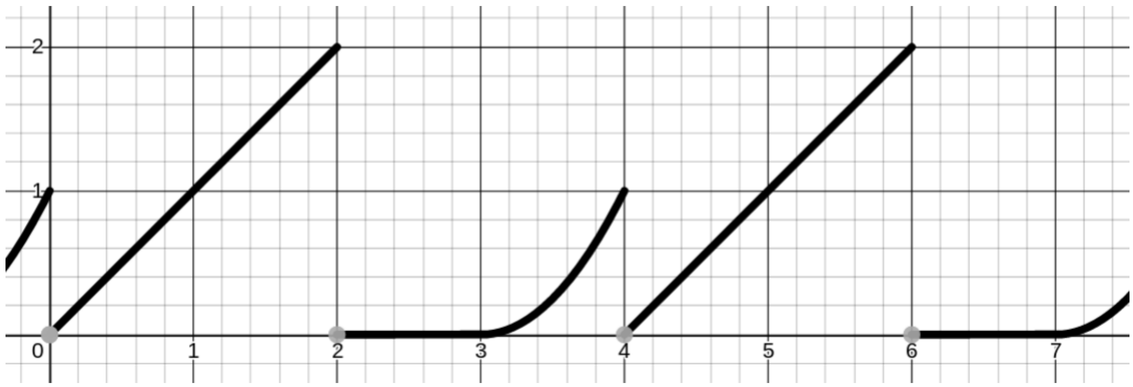
(d) A spiral sink if $L$ is small, and a nodal sink if $L$ is large

(e) A nodal sink if $L$ is small, and a spiral sink if $L$ is large

(f) None of the above

2. Suppose that the piecewise smooth function shown has the Fourier series

$$f(t) \sim \frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos(n\pi t/2) + b_n \sin(n\pi t/2)\right).$$




What is $\frac{a_0}{2} - a_1 + a_2 - a_3 + a_4 - \cdots$?

(a) 1

(b) $-1$

(c) $\pi^2/6$

(d) 0

(e) 1/2

(f) None of the above

3. Let $F$ be the odd 4-periodic function such that

$$F(t) = \begin{cases} 0 & 0 \le t \le 1 \\ t-1 & 1 \le t < 2. \end{cases}$$

What is the 1000th coefficient in the Fourier series for $F$?

(a) $1/500\pi$

(b) $(4 - 2000\pi)/1000^2\pi^2$

(c) $(4 + 2000\pi)/1000^2\pi^2$

(d) $0$

(e) $1/2000\pi^2$

(f) None of the above

4. An undamped spring-mass system with mass 2 kg and spring constant 5 kg/s$^2$ is forced by the 4-periodic sawtooth wave given by $F(t) = 2t$ for $t \in (-2, 2)$. $F$ is measured in Newtons and $t$ is measured in seconds. If the mass also moves with period 4, which of the following is the best approximation to the amplitude of its motion in meters?

(a) $\frac{8}{5\pi - 0.5\pi^3}$

(b) $\frac{8}{4\pi^3 - 10\pi}$

(c) $\frac{8\pi - 8}{5\pi^2 - 0.5\pi^4}$

(d) $\frac{8}{5\pi}$

(e) $\frac{8}{\pi - 2\pi^2}$

5. A 1-meter aluminum bar with thermal diffusivity 1 $\text{cm}^2/\text{s}$ is insulated around the sides so that no heat can escape, and its two ends are held at absolute zero. At time $t = 0$ s, half of the bar is at absolute zero and the other half is at 400 K. What is the exact temperature (in K) at the bar's midpoint at $t = 1000$ s?

   (a) 0
   (b) 200
   (c) $800 \sum_{n=1}^{\infty} \frac{1}{\pi n} \sin(\pi n/2) \exp(-\pi^2 n^2)$
   (d) $800 \sum_{n=1}^{\infty} \frac{1}{\pi n} \sin^2(\pi n/4) \sin(\pi n/2) \exp(-\pi^2 n^2)$
   (e) $800 \sum_{n=1}^{\infty} \frac{1}{\pi n} \sin^2(\pi n/4) \sin(\pi n/2) \exp(-\pi^2 n^2/10)$
   (f) None of the above

6. Find the steady-state solution to the following differential equation:

$$x'' + 2x' + 3x = 1 + \sum_{n=1}^{\infty} \frac{1}{n^2} \cos(nt).$$

(a) $x = C_1 e^{-t} \cos(\sqrt{2}t) + C_2 e^{-t} \sin(\sqrt{2}t)$

(b) $x = \sum_{n=1}^{\infty} \frac{1}{n^2} \left( \frac{3-n^2}{(3-n^2)^2+4n^2} \cos(nt) + \frac{2n}{(3-n^2)^2+4n^2} \sin(nt) \right)$

(c) $x = \sum_{n=1}^{\infty} \frac{1}{n^2((3-n^2)^2+4n^2)} \cos(nt)$

(d) $x = \sum_{n=1}^{\infty} \frac{\cos(nt-\pi n/2)}{n^2\sqrt{(3-n^2)^2+4n^2}}$

(e) None of the above

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