Practice Midterm 1 MA 35100

Name:

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There are 10 points for each problem. The first two problems have to be answered in a written response and a complete argument has to be presented. For the following 8 multiple choice problems, the 10 points are achieved if the correct response is indicated on this page. Up to 5 points can be achieved if a written response on the page of the problem contains essential ideas how to solve the problem.

Please fill in your answers in the following table:

	(a)	(b)	(c)	(d)	(e)
(3)	☐	☐	☐	☐	☐
(4)	☐	☐	☐	☐	☐
(5)	☐	☐	☐	☐	☐
(6)	☐	☐	☐	☐	☐
(7)	☐	☐	☐	☐	☐
(8)	☐	☐	☐	☐	☐
(9)	☐	☐	☐	☐	☐
(10)	☐	☐	☐	☐	☐

1. Let V be a vector space. Give the definition of the dimension of V.

(Continue your answer on the back of the page, if needed.)

2. Let A be a 4×2 matrix. Suppose that there is a vector $B = [b_1, b_2, b_3, b_4]^T$, such that

$$A \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = B$$

has a 2-dimensional solution set. Is it true that $AX = C$ always has at least one solution, for every $C = [c_1, c_2, c_3, c_4]^T$? Explain why or why not.

(Continue your answer on the back of the page, if needed.)

3. What is a basis of the null space of

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}?$$

(a) $\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$

(b) $\begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$

(c) $\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$

(d) $\begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$

(e) $\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$

(Continue your answer on the back of the page, if needed.)

4. What is true about the matrix

$$A = \begin{bmatrix} 0 & 3 & 2 \\ 2 & -7 & 4 \\ 4 & -8 & 12 \\ 5 & -4 & 19 \end{bmatrix}?$$

(a) The dimension of its column space is 1.

(b) The rank of the transpose of A equals the nullity of the transpose of A.

(c) The rank of A equals the nullity of A.

(d) The nullspace of A has dimension 2

(e) The column space is $\mathbb{R}^4$.

(Continue your answer on the back of the page, if needed.)

5. What is true about the linear system

$$\begin{aligned} x + 2z &= 3 \\ y + 2z &= 3 \\ x + y + 4z &= 5 \end{aligned}$$

(a) There is no solution.

(b) There is exactly one solution and $x = 1$.

(c) There is exactly one solution and $y = 2$.

(d) There are infinitely many solutions and $x = 3 - 2z$.

(e) There are infinitely many solutions and $z = 0$.

(Continue your answer on the back of the page, if needed.)

6. Which of the following sets of vectors span $\mathbb{R}^3$?

(a) $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$

(b) $\begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ 0 \\ 2 \end{bmatrix}$

(c) $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

(d) $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$

(e) $\begin{bmatrix} 0 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 5 \\ -4 \\ -6 \end{bmatrix}$

(Continue your answer on the back of the page, if needed.)

7. Which of the following sets of vectors is *not* a basis of

$$\text{span}(\begin{bmatrix} 2 \\ 0 \\ 1 \\ 10 \end{bmatrix}, \begin{bmatrix} 3 \\ 0 \\ 1 \\ 14 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ 3 \\ 14 \end{bmatrix})?$$

(a) $\begin{bmatrix} 2 \\ 0 \\ 1 \\ 10 \end{bmatrix}, \begin{bmatrix} 3 \\ 0 \\ 1 \\ 14 \end{bmatrix}$

(b) $\begin{bmatrix} 1 \\ 0 \\ 0 \\ 4 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 2 \end{bmatrix}$

(c) $\begin{bmatrix} 2 \\ 0 \\ 1 \\ 10 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ 3 \\ 14 \end{bmatrix}$

(d) $\begin{bmatrix} 1 \\ 0 \\ 0 \\ 3 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 3 \end{bmatrix}$

(e) $\begin{bmatrix} 3 \\ 0 \\ 1 \\ 14 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ 3 \\ 14 \end{bmatrix}$

(Continue your answer on the back of the page, if needed.)

8. Which is a subspace of $\mathbb{R}^2$?

(a) $\{\begin{bmatrix} x \\ y \end{bmatrix}$ such that $x^2 = 0\}$

(b) $\{\begin{bmatrix} x \\ y \end{bmatrix}$ such that $2x + 3y = 5\}$

(c) $\{\begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix}\}$

(d) $\{\begin{bmatrix} a+1 \\ b+c \end{bmatrix}$ such that $a, b, c \in \mathbb{R}\}$

(e) $\{\begin{bmatrix} x \\ y \end{bmatrix}$ such that $x^2 = y^2\}$

(Continue your answer on the back of the page, if needed.)

9. What is

$$\begin{bmatrix} 2 & 3 & 1 \\ -1 & 5 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}?$$

(a) $\begin{bmatrix} -1 \\ -2 \\ 1 \end{bmatrix}$

(b) $\begin{bmatrix} 2 & -3 & 0 \\ -1 & -5 & 0 \end{bmatrix}$

(c) $\begin{bmatrix} 2 & -3 \\ -1 & -5 \end{bmatrix}$

(d) $\begin{bmatrix} -1 \\ -6 \end{bmatrix}$

(e) It is not defined.

(Continue your answer on the back of the page, if needed.)

10. Which of the following is **not** a basis of the vector space of polynomials of degree at most 2?

    (a) $1, x+1, x^2+x+1$

    (b) $x+2, x^2-x+1, x^2+x+5$

    (c) $1, x, x^2$

    (d) $x+1, x+2, x^2$

    (e) x^2+x+1, x^2-2x+1, x^2-2

(Continue your answer on the back of the page, if needed.)