MA 57300. NUMERICAL SOLUTIONS OF DIFFERENTIAL EQUATIONS (FALL 2025)

INSTRUCTOR: DI QI (EMAIL: QIDI@PURDUE.EDU)

Time: Tuesday, Thursday 10:30 AM - 11:45 AM, MJIS 1083
First Class: Mon., August 25, 2025
Last Class: Sat., December 13, 2025

For other important dates, see Purdue 2025-2026 Academic Calendar.
(see also *`Course Webpage`* for supplementary course materials)

Course Description:

This course meant to introduce graduate students with various background to the fundamentals and applications of numerical methods essential for solving differential equations and dynamical systems. The course will cover key concepts with a balance between rigorous analysis and numerical implementation. Solutions of ordinary differential equations will be discussed, including single and multistep methods for initial value problems, and Runge-Kutta schemes, iterative methods for solving large systems of equations, convergence and stability, and methods for stiff problems such as exponential temporal integrators and multigrid iterative solvers. Theories and implementations for numerical solutions to stochastic differential equations and sequential Monte Carlo sampling strategies will then be explored. Numerical methods for dynamical systems will be discussed including Hamiltonian systems, invariant sets and chaotic attractors. The course will continue to applications in finance using the time stepping schemes to solve boundary and eigenvalue problems of partial differential equations, and data-driven strategies for solving time-dependent sequential predictions.

Prerequisite: The students are expected to have a good background in calculus, linear algebra, complex variables, and elementary background of ODE and PDE. Some experience with writing computer programs (in MATLAB, Python or another language) is also recommended but not necessary.

Audience: The course should be suitable to any graduate student in applied and computational mathematics, physics, engineering, as well as related fields involving numerical computing and computer programming.

Major Learning Outcomes:

(1) Numerical solutions of initial-value problems, general one-step methods;
(2) Explicit and implicit Runge-Kutta and multistep methods, convergence and stability;
(3) Iterative methods for solving large systems of equations;
(4) Numerical methods for stochastic differential equations and Monte Carlo methods;
(5) Numerical methods for Hamiltonian systems, invariant sets and attractors;
(6) Using software tools such as visualization and specialized computing packages.

Learning Resources & Course Arrangements:

Reference books and reading materials:

- "Dynamical Systems and Numerical Analysis" by A. M. Stuart and A. R. Humphries
- "Solving Ordinary Differential Equations I: Nonstiff problems, and II: Stiff and differential-algebraic problems" by E. Hairer, S. P. Nørsett, and G. Wanner.
- "Numerical Methods of Stochastic Differential Equations" by P. E. Kloeden and E. Platen
- "Monte Carlo Strategies in Scientific Computing" by Jun S. Liu
- "A First Course in The Numerical Analysis of Differential Equations" by Arieh Iserles
- "Spectral Methods in Matlab" by Nick Trefethen

Course Resources: The main learning management system will be *Brightspace.* Lecture notes and homework will be posted on *Brightspace*. The following platforms (accessible through *Brightspace*) will also be used for assignments and other activities:

- *Gradescope* for homework/project submission.
- *Piazza* for class discussions.
- *Zoom* for potential online discussions and virtual meetings.

Course Schedule:

Week	Start Date	Topics	Comments
1	Aug. 25	Introduction, Euler's method	
2	Sep. 1	initial value problems	
3	Sep. 8	multi-step methods	
4	Sep. 15	zero stability and convergence	
5	Sep. 22	absolute stability	
6	Sep. 29	stiff problems	
7	Oct. 6	iterative methods	
8	Oct. 13	boundary and eigenvalue problems	No class on Oct. 14 (October break)
9	Oct. 20	stochastic differential equations	
10	Oct. 27	stochastic time differentiation	
11	Nov. 3	Monte-Carlo sampling of SDEs	
12	Nov. 10	invariant sets and equilibrium measure	
13	Nov. 17	Hamiltonian and conservative systems	
14	Nov. 24	Conservative systems (continued)	No class on Nov. 27 (Thanksgiving vacation)
15	Dec. 1	Symplectic schemes	
16	Dec. 8	Students presentation	

(Schedule is subject to change. Any changes will be posted on *Brightspace*.)

Lecture Notes: Lecture notes will be posted in *Brightspace* and will be organized in form of chapter modules (Chapter 1, Chapter 2, etc). Typically, there will be two lectures per week, each corresponding to a section in the textbook and about 75 mins long.

Office Hours: TTh, 1:00-2:00 pm (Eastern Time) or through appointment. The time is subject to change. The information will be posted on *Brightspace*.

Assignments and grading: There will be project assignments during the semester, which will involve analysis, coding, analyzing/plotting results, and preparing a typeset report (preferably in LaTex). You may discuss strategies to solve the problems with peers, but every line of code and everything in the solution (including figures) for each homework must be 100% yours. Allowing others to copy code or solutions from you is considered cheating. AI tools can be used to help learning and understanding but is not allowed in any part of homework and project.

The students are expected to present their results in the last week of the class. There will be no midterm and final exams. The grade will be based on the project and attendance (60%) and final presentation (40%).

This semester, we will apply the following rule: students who get at least 97% of the total points are guaranteed an A+, 93% guarantees an A, 90% an A-, 87% a B+, 83% a B, 80% a B-, 77%a C+, 73% a C, 70% a C-, 67% a D+, 63% a D, and 60% a D-. Please note, these are not the actual cutoffs, but rather upper bounds on those. The actual cutoffs of these grades will be determined after the final exam and can be lower but not higher than the ones above. Thus, the actual cutoff for A can be, say 85%, but not 95%.

Course and University Policies:

Attendance and course engagement: This is an in-person course. The students are expected to attend the lectures and read the corresponding lecture notes. Students are responsible for completing and submitting all assignments on time. Students need to give presentation on their project in the last week of the semester. Students are encouraged to participate actively during the lectures and after-class discussions. Students must also check periodically for possible changes in the course schedule (on *Brightspace*) including due dates for assignments and exams.

Makeup policy: Student needs to inform the instructor of any conflict that can be anticipated and will affect the submission of an assignment or the ability to give the presentation. Only the instructor can excuse a student from a course requirement or responsibility. When conflicts can be anticipated, such as for many University-sponsored activities and religious observations, the student should inform the instructor of the situation as far in advance as possible. For unanticipated or emergency conflict, when advance notification to an instructor is not possible, the student should contact the instructor as soon as possible by email, through *Brightspace*, or by phone. When the student is unable to make direct contact with the instructor and is unable to leave word with the instructor's department because of circumstances beyond the student's control, and in cases of bereavement, quarantine, or isolation, the student or the student's representative should contact the Office of the Dean of Students via email or phone at 765-494-1747. Our course *Brightspace* includes a link on Attendance and Grief Absence policies under the University Policies menu.

Academic integrity: Academic integrity is expected for all students at all times in this course. You are free (even encouraged) to work with other students to solve the homework problems. However, you are required to complete and write up solutions for the homework using your own words and on your own. If you worked with any humans, book, the internet, you should be explicit about it and list all sources and the extent of help you got from each resource (no points will be taken for such disclosures). But if you present as your own work what was not, then you will get zero points on the assignment and an academic misconduct filing after the first instance.

Nondiscrimination statement: Purdue University is committed to maintaining a community which recognizes and values the inherent worth and dignity of every person; fosters tolerance, sensitivity, understanding, and mutual respect among its members; and encourages each individual to strive to reach his or her own potential. In pursuit of its goal of academic excellence, the University seeks to develop and nurture diversity. The University believes that diversity among its many members strengthens the institution, stimulates creativity, promotes the exchange of ideas, and enriches campus life. More details are available on our course *Brightspace* table of contents, under University Policies.

Students with disabilities: Purdue University strives to make learning experiences accessible to all participants. If you anticipate or experience physical or academic barriers based on disability, you are encouraged to contact the Disability Resource Center at: drc@purdue.edu or by phone: 765-494-1247, as soon as possible. If the Disability Resource Center (DRC) has determined reasonable accommodations that you would like to utilize in this class, you must send your Course Accommodation Letter to the instructor. Instructions on sharing your Course Accommodation Letter can be found by visiting: https://www.purdue.edu/drc/students/course-accessibility-letter.php https://www.purdue.edu/drc/students/course-accommodation-letter.php. Additionally, you are strongly encouraged to contact the instructor as soon as possible to discuss implementation of your accommodations.

Emergency preparation: In the event of a major campus emergency, course requirements, deadlines and grading percentages are subject to changes that may be necessitated by a revised semester calendar or other circumstances beyond the instructor's control. Relevant changes to this course will be posted onto the course website or can be obtained by contacting the instructors or TAs via email or phone. You are expected to read your *@purdue.edu* email on a frequent basis.

COVID-19 statement: If you become quarantined or isolated at any point in time during the semester, in addition to support from the Protect Purdue Health Center, you will also have access to an Academic Case Manager who can provide you academic support during this time. Your Academic Case Manager can be reached at acmq@purdue.edu and will provide you with general guidelines/resources around communicating with your instructors, be available for academic support, and offer suggestions for how to be successful when learning remotely. Importantly, if you find yourself too sick to progress in the course, notify your academic case manager and notify me via email or *Brightspace*. We will make arrangements based on your particular situation. The Office of the Dean of Students (odos@purdue.edu) is also available to support you should this situation occur. Other important policie scan be found in *Brightspace*.

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