# LINKING COLLEGE PRE-CALCULUS STUDENTS' USES OF GRAPHING CALCULATORS TO THEIR UNDERSTANDING OF MATHEMATICAL SYMBOLS

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*This study examined ways in which students make use of a graphing calculator and how use relates to comfort and understanding with mathematical symbols. Analysis involved examining students' words and actions in problem solving to identify evidence of algebraic insight. Findings suggest that lack of symbol sense can lead students to turn to a graphing calculator as a tool for prompting a way to start a problem, or for providing a guess or confirmation. Certain symbols also lead some students to believe that they cannot use a calculator at all. Implications for teaching with a graphing calculator are included.*

## Introduction

Students often have access to graphing calculators and use them to help solve many types of problems. However, teachers and researchers are often unaware of how and why students use graphing calculators and how their use relates to their mathematical thinking, particularly about mathematical symbols. In this study, I address the research question: How are students' uses and understandings of graphing calculators related to students' uses and understandings of symbols?

Symbols are components of mathematical language that allow a person to communicate, manipulate, and reflect upon abstract mathematical concepts. However, symbolic language is often a cause of great confusion for students (Rubenstein & Thompson, 2001). Expert mathematicians or teachers are able to manipulate and to interpret mathematics through its symbolic representations, whereas students may struggle in this endeavor; they often need to be told what to see and how to reason with mathematical symbols (Bakker, Doorman, & Drijvers, 2003; Kinzel, 1999; Stacey & MacGregor, 1999). Arcavi (1994) explains that working fluently with symbols in mathematics requires developing strong *symbol sense* which includes, for example, understanding when to call on or abandon symbols in problem solving, understanding the need to compare symbols meaning with one's own expectations and intuitions, and knowing how to choose possible symbolic representations. Arcavi sees development of symbol sense as a necessary component of general sense making in mathematics. It is a tool that allows students to read into the meaning of a problem and to check the reasonableness of symbolic expressions.

Difficulties with symbol manipulation in mathematics may be one reason that students turn to graphing calculators for assistance in problem solving. Unlike calculators with computer algebra system (CAS) capabilities, most graphing calculators (e.g., TI-83, TI-84, TI-85, Casio FX-9750) cannot algebraically manipulate symbolic equations to produce useful results. Some work with symbols can be done with a non-CAS calculator; for example, symbolic expressions can be entered and viewed in the *Y=* menu, and values can be stored as a variable and substituted into an expression. However, a large benefit of these tools is that users can explore other representational forms of symbolic expressions, such as graphs, tables, or matrices.

## Framework

Pierce and Stacey (2001) used Arcavi's (1994) notion of symbol sense to develop a conceptual framework for looking at what is needed to take a mathematical problem, work with it using the tools and language of the calculator, and interpret and use the results using regular mathematical notation and forms. Pierce and Stacey define *algebraic insight* as a subset of symbol sense that enables a learner to interact effectively with a computer algebra system (CAS) when solving problems. They suggest that the nature of algebraic insight is the same whether work is done by-hand or with a CAS. Thus, I contend that the framework for assessing algebraic insight can also be appropriate for examining students' problem solving with graphing calculators that do not have symbolic manipulation capabilities. The two components that make up algebraic insight, *algebraic expectation* and the *ability to link representations*, elaborate instances and examples of algebraic insight that may be identifiable when analyzing students' work with graphing calculators. Table 1 shows the elements of the algebraic insight framework.

Table 1
*Algebraic Insight Framework (Pierce & Stacey, 2001)*

| **Aspects** | **Elements** | **Common Instances** |
|---|---|---|
| Algebraic Expectation | 1.1 Recognition of conventions and basic properties | 1.1.1 Know meaning of symbols<br>1.1.2 Know order of operations<br>1.1.3 Know properties of operations |
| | 1.2 Identification of structure | 1.2.1 Identify objects<br>1.2.2 Identify strategic groups of components<br>1.2.3 Recognize simple factors |
| | 1.3 Identification of key features | 1.3.1 Identify form<br>1.3.2 Identify dominant term<br>1.3.3 Link form to solution type |
| Ability to Link re-presentations | 2.1 Linking symbolic and graphic reps | 2.1.1 Link form to shape<br>2.1.2 Link key features to likely position<br>2.1.3 Link key features to intercepts and asymptotes |
| | 2.2 Linking symbolic and numeric reps | 2.2.1 Link number patterns or type to form<br>2.2.2 Link key features to suitable increment for table<br>2.2.3 Link key features to critical intervals of table |

This framework specifically addresses ways to plan, assess, and reflect on students' understanding when working with technology to solve mathematical problems (Pierce & Stacey, 2001). Using this framework assists in the task of identifying ways in which students' uses and understandings of mathematical symbols relate to how and why they use a graphing calculator.

## Methods

The method of inquiry for this research is a multi-case study, where a case represents an individual college pre-calculus student. Students were selected for this study using a survey that assessed familiarity with and use of graphing calculators. All invited participants indicated having at least average familiarity with graphing calculators and using graphing calculators at least one-half of the time on homework, but reported varying levels of success in previous mathematics course. Six students agreed to participate in the study and have the pseudonyms: Jill, Nina, Molly, Beth, Elyse, and Shawn.

The data sources in this study include a collection of work on tasks, video recordings of interviews, and computer recordings of calculator keystrokes for work completed on a graphing calculator. For the latter, I connected TI-84+ graphing calculators to a computer via a TI-Presenter device, and Windows Movie-Maker software captured and recorded videos of students' calculator keystrokes (c.f. McCulloch, 2007).

The findings reported in this paper are one piece of a larger doctoral thesis study. Students participated in three different interview settings during the course of the larger study. The results in this paper come from individual task-based interviews that took place near the beginning of the semester. In these sessions, students worked on secondary school-level algebra problems (i.e., problems to which students should have had prior exposure, but which had not recently been covered in class). As they worked on four different tasks (given in Table 2), students talked aloud about their thoughts and actions. They shared reasons for making use of a graphing calculator and discussed the specific activities that they were employing. If a student did not use the graphing calculator at all, I asked them to discuss why it was not useful on the problem and to consider ways in which they could have used it.

Table 2
*Interview Tasks*

| |
|---|
| Task 1 – Solve a rational equation: $\frac{x-16}{x^2-3x-12}=0$ |
| Task 2 – Solve a polynomial equation: $x^3+2x-4=8$ |
| Task 3 – Setup and solve a linear word problem: A theater manager sold 5200 tickets and the receipts totaled $32,200. The adult admission is $8.50, and the children's admission is $6.00. How many adult patrons were there? |
| Task 4 – Solve a linear inequality: $\left\|3x-\frac{2}{7}\right\|+1.2>5$ |

Transcriptions of videos of work on graphing calculators made it easier to follow students' work with this tool. I began by assigning basic codes indicating the manner of calculator use to the lines of the transcripts to help identify students' uses of the tool (e.g., graphed, used a table, computed, etc). I then looked in-depth at students' words and actions when using a graphing calculator and identified elements of symbol sense that were both evident and lacking in students' work by using the algebraic insight framework.

## Findings

Findings are organized around the four interview tasks. Each of the following subsections explains specific ways in which students engaged with the graphing calculator in activities. Instances of algebraic insight indicate details about students' symbol sense in their work.

### *Task Type: Solve a Rational Equation*

Given the rational equation, $\frac{x-16}{x^2-3x-12}=0$:

- Jill and Molly graphed the numerator and denominator separately;
- Molly looked at a table to find the $y$-value when $x$=0;
- Elyse used the graphing calculator for computation only;

- Jill tried to see if linear or quadratic regression could work;
- Molly typed the left side of the equation into the main calculator screen to see if the calculator would "breakdown" the problem or solve for $x$;
- Beth, Nina, and Shawn did not use the graphing calculator at all.

These activities suggest that some students had the symbol sense to know that it was possible to abandon symbolic manipulation. However, the specific ways in which they used the graphing calculator indicate a lack of understanding of what the calculator could do. For example, Molly started by typing a function into the main screen, hoping that the calculator could "give a break down" of the problem. She explained, "The calculator is not simplifying for $x$ here. It's not solving for $x$. There's a way to solve for $x$, isn't there?" It seems that she remembered using a graphing calculator in the past to solve for $x$, but could not remember what to do.

Molly and Jill chose to graph the numerator and denominator as separate graphs. This suggests a lack of algebraic insight for linking the rational form to the shape of the graph, which might not be surprising if they did not have experience with graphing rational functions. The linear form of the numerator and quadratic form of the denominator may have been more familiar and may be forms that they could easily link to shape and know what to expect in the graph. However, after graphing the two functions, neither student knew how to use the graphs to solve the given question. Molly looked at a table and found values when $x$ equaled zero, saying, "I'm hoping that the calculator will be able to tell which one of the equations I'm supposed to use." Jill abandoned the graphs and attempted to see if linear regression would be useful instead.

Other students gave reasons for not using the calculator here. Beth explained that it could not tell her what steps to follow, saying, "It doesn't tell the mechanics of the problem that you have to do. It just gives you a number." Elyse expressed similar frustration, saying, "I don't know of a way where you can put an $x$ in. The only thing I can think of is if you substituted something for the $x$ maybe." Both Elyse and Beth were comfortable using the calculator for numeric calculations, but struggled to understand how to use the tool with algebraic symbols.

### *Task Type: Solve a Polynomial Equation*

Given the polynomial equation, $x^3 + 2x - 4 = 8$, students used a graphing calculator in the following ways:

- Beth graphed only the left side of the equation
- Molly graphed the function on the left side and evaluated at $x$=8;
- Nina and Beth set the equation equal to zero and graphed to evaluate at $x$=0;
- Beth, Molly, and Shawn used the calculator for computations;
- Nina and Shawn used a table to determine the $x$-value when $y$ equaled zero;
- Shawn set the equation equal to zero and looked at the graph to see how many $x$-intercepts existed;
- Jill and Elyse did not use the graphing calculator at all.

Nina and Shawn exhibited evidence of algebraic insight for linking key features to intercepts and to critical intervals for a table. However, both students only chose to use the graphing calculator after prompting from the researcher. Nina admitted that she would never have chosen to use a table to solve the problem on her own, and Shawn had already found an answer on paper and only used the calculator because he was not completely satisfied with his answer.

Shawn and Nina did not entirely trust the answers they found using the calculator. Shawn noticed that the graph only crossed the $x$-axis one time, and found one answer of $x$=2 from the table, but he had found three different answers on paper. Nina also anticipated finding three answers to the problem and expected that there were more answers than the calculator was telling her. This anticipation for a certain number of answers connects to students' algebraic insight for linking the problem form to the solution type and linking symbol meaning to prior experiences. Shawn also exhibited symbol sense for linking key features to critical intervals of a table when he noticed that two of his answers on paper had been decimal values and the table he was using only counted by integers. Neither student, however, made a clear link between the symbolic and graphical representations, which provided strong evidence that there was only one zero for the function $x^3 + 2x - 12$.

Other students struggled with the meaning of symbols and with identifying the dominant term needed for finding the solution (e.g., looking at $x$=0 instead of $y$=0). Molly misinterpreted the meaning of the equation and felt that she was supposed to substitute the value 8 in for $x$. Her trust in this interpretation allowed her to ignore the fact that the calculator produced a result of $y$=524, even though she had anticipated finding a value for $x$. Nina performed a similar action by evaluating the function at $x$=0 to find the zero. However, the calculator's result of $y$=-12 caused her to realize her mistake and change her activity to find $x$ instead of $y$.

*Task Type: Setup and Solve a Linear Word Problem*

Students used a graphing calculator sparingly on Task 3 in the following ways:

- Jill tried to graph an equation with two variables, but stopped when she could not determine how to enter both variables;
- Molly graphed two equations with the same variable;
- Beth, Nina, and Elyse used computations to make sense of the information;
- Molly, Jill, and Shawn computed values to find an answer;
- Nina used computation to see if her equation made sense.

Molly and Jill used the calculator as a numeric tool that could help them abandon symbolic manipulation for a guess and check strategy. Jill had created two useful symbolic equations with two variables and was using the calculator to guess and check instead of solving the equations simultaneously, while Molly was unable to create an equation. With symbolic forms for reference, Jill was able to continually link her results to the meaning of the symbols in the problem, while Molly lost sight of key information and was not able to reach a solution.

Beth, Nina, and Elyse struggled with the symbol sense to select or create possible symbolic representations, and used the calculator in the hopes of discovering a useful relationship in the given information. For example, Elyse tested to see if all of the tickets could be adult tickets by dividing 32200 by 8.50, but was disappointed to get a decimal answer. She explained, "Maybe if it divided evenly, I may believe it was only adults." Nina and Beth both used a similar calculation, but also divided 32200 by 6.00 to see if this value divided evenly. Beth continued dividing all given values by each other in the hopes of finding a number that might work as an answer to the problem. These students tried to manipulate numbers on the calculator to answer the problem and avoid the need for creating symbolic equations.

*Task Type: Solve a Polynomial Inequality*

Due to time constraints in the study, only four of the participants worked on the following inequality problem: Solve for $x$ given $\left|3x - \frac{2}{7}\right| + 1.2 > 5$. Students used the graphing calculator as follows:

- Jill, Beth, Nina, and Elyse used the calculator for computations;
- Beth used the calculator to convert decimals and integers into fractions;
- Elyse used the calculator to check a hypothesis.

Beth answered this problem by plugging one number in and using her calculator to compute a value on the left and seeing that the result was greater than five. She made comments such as, "Seeing that that (on paper) and this (on screen) looks the same, then you think, well I must be doing something right," suggesting that she did not realize that copying the direct computations from the calculator was not the same as doing the problem by hand. In this case, she did not recognize her dependence on the calculator for helping her manipulate the values on paper. The other three students manipulated the problem as though it was a linear equation, sometimes paying attention to the sign inside the absolute value symbol or the direction of the inequality. None of the students demonstrated algebraic insight for linking the form of this problem to a proper solution type, and their actions were restricted to numeric manipulations suggested by the operation signs in the problem. They used the calculator for calculating with fractions and decimals only.

When asked if a graphing calculator could be useful for this problem in other ways than as a computational tool, students responded in a variety of ways. Beth said that she did not know of a way to use it. Nina said she could not use it because she did not know how to handle the inequality sign, while Jill said she did not recall how to input absolute value. Elyse answered, "It's algebra so you can't just plug the whole thing in and get your answer." Thus, three of these students identified particular symbols in the problem (absolute value, inequality sign, and the $x$-variable) as the reason for not using the graphing or table features of the calculator. At the same time, numeric symbolic structures such as fractions and decimals were identified by all four students as important reasons for needing the calculator for computations.

## Discussion

A detailed examination of students' use of graphing calculators and what they said while using them can provide insight into the relationship between students' understanding of symbols and understanding of graphing calculators. By looking closely at specific details surrounding students' graphing calculator use, I identified two themes that address the research question:

### *1. Lack of Symbol Sense Caused Students To Use a Graphing Calculator For Help.*

Students had some dependence on the graphing calculator as a tool for abandoning symbolic manipulation and finding an answer or a procedure to follow. At times, students treated the tool as a partner (Goos, Galbraith, Renshaw, & Geiger, 2003) that could help in the following ways: (a) by providing directions or a prompt, (b) by providing an accessible answer, and (c) by providing confirmation. The following paragraphs illustrate these categories of use.

Students often had difficulty trying to decide how to start a problem. In these instances, they sometimes turned to the graphing calculator to prompt an activity. For example, Molly wanted the calculator to tell her how to break down the rational equation in the initial interview. She entered the function into the main screen in the hopes that it would tell her something about the manipulations needed for the problem. She also graphed the numerator and denominator of the rational equation, saying that she was hoping for the calculator to tell her which function to use. She turned to the calculator for directions on how to solve the problems.

In some situations, students tried to avoid working with the symbols on paper and worked on the graphing calculator to try to find an answer. For example, Beth, Nina, and Elyse sought answers from the calculator on the linear word problem when they divided different given numbers in the hopes some value would divide evenly into another. They struggled to create symbolic equations for the problem, and tried to avoid a need for symbols by seeking an easy, familiar looking numeric solution from the calculator.

Some students recognized that a calculator was useful for confirming an idea or checking the reasonableness of an answer. For example, while working on a polynomial equation, Molly, Nina, and Shawn all used graphs and tables to find an answer to the problem and compared the calculator's answers with work they had done on paper. This caused problems when calculator answers did not match their expectations, and caused some students to mistrust the calculator.

*2. Lack of Understanding of a Graphing Calculator's Abilities to Handle Symbolic Forms Kept Students From Using Them or Using Them Correctly.*

Many of the students had difficulty knowing when and how to interact with a graphing calculator when solving symbolic problems. Difficulties were due to both misunderstanding of the technology and misconceptions about the mathematics involved in the problem. One fact that was evident from students' work is that they often did not know many of the features that the technology offered. For example, Beth and Elyse insisted that they could not use the graphing calculator when there was a variable in the equation. Similarly, Jill did not think she could enter an absolute value sign on a calculator, and Nina did not think there was a way to work with inequalities on the calculator. None of the students seemed to be familiar with menu options such as MAXIMUM or ZERO or INTERCEPT when working with graphs. Most of the students chose to use TRACE to find points on a graph instead, which does not provide exact values for answering a question. The students often struggled to see a use for the graphing calculator in problem solving because they were not aware of the powerful options it provided.

## Implications and Conclusions

Students were uncomfortable with and not proficient with using graphing calculators, despite their claims for being so on the initial survey. However, these students still had a certain amount of dependence on the graphing calculator for helping them postpone or abandon symbolic manipulation when it was causing them trouble. The fact that graphing calculators provide students with a way to do mathematics without using algebraic manipulation techniques has been identified in the research as a reason that some teachers give for opposing calculator use, especially at the college level (Hennessy, Fung, & Scanlon, 2001). However, other researchers have found that students' use of a calculator in this way is critical for helping them explore a problem to consider expectations before attempting an analytical solution (Quesada & Maxwell, 1994). Data from this study suggests that graphing calculators could be especially useful with weaker students (such as those taking or retaking college pre-calculus) as a tool for helping students gain more experience with important mathematical symbols and concepts. The teachers in this study did not teach or assess with graphing calculators and, consequently, restricted classroom examples and test questions to easy functions (e.g. no fractional coefficients, quadratic functions that could be factored, etc.). This practice may increase students discomfort with less used symbols such as inequality signs, absolute values, fractions, square roots and high powers of $x$. Many mathematical problems cannot or should not be solved by hand, but the students in this study did not seem aware of this possibility (e.g. the polynomial inequality in Task 2 was best solved using a calculator, but all students expected there to be an accessible pen and paper solution method). Awareness and understanding of how a graphing calculator can serve a student's needs when encountering mathematics inside and outside of the mathematics classroom is an important part of teaching mathematics, especially at the college level.

When students have access to a graphing calculator, and do not know how to use it or do not understand or remember what it is capable of doing, they can use it in creative and inefficient ways. Gray and Tall (1994) suggest that students who did not have a strong understanding of the different uses of symbols may develop different, incorrect techniques for problem solving due to their personal interpretations of the symbols. The same idea may apply to students who do not have a strong understanding of how and when to use a graphing calculator. Teachers need to be aware of some of the non-standard uses that students can create to seek assistance from a graphing calculator as they try to avoid or abandon symbolic manipulation.

For the students in this study, understanding how to work with mathematical symbols on paper had a connection to their choices of how and when to use a graphing calculator. However, students demonstrated limited algebraic insight for linking representations and connecting what they were doing on the calculator to their work with symbols on paper. When teaching with a graphing calculator, teachers must be careful not to treat the tool as a different way of approaching a problem, but instead integrate it into a problem and help students reflect on how the work displayed on the screen relates to the symbols on paper.

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